Toward partial compositeness on the lattice: Lattice results from a candidate SU(4) gauge theory

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With the Tel Aviv-Colorado (TACo) Collaboration

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Outline

- Birds-eye view / motivation
- Section 1: Phenomenology
- Section 2: Meson Spectroscopy
- Section 3: The Higgs Potential
- Section 4: Baryons
- Summary / conclusions / future directions

The birds-eye view

- This talk is about gauge-fermion systems with fermions charged under multiple different representations of the gauge group:

$$\text{``}\mathcal{L} = F^2 + \sum_r \bar{\psi}_r D_r \psi_r\text{''}$$

- These systems are possibly relevant for phenomenology and definitely interesting from a quantum field theory perspective

- My colleagues and I are conducting lattice simulations of one such system

Motivation

- Experimentally, physics exists beyond the Standard Model:
  - Neutrino masses (if SM is an effective theory, still need a UV completion)
  - Dark matter
  - Dark energy
  - (Quantum gravity?)

- Theoretically, the Standard Model is not entirely satisfactory
  - What is the origin of the Higgs potential? Why is the Higgs mass 125 GeV?
  - What is the origin of observed hierarchies? For instance, why is $m_t / m_{u/d} \sim 10{,}000$?

- Theoretically, our understanding of strongly coupled QFT remains incomplete.
  - In particular, which mechanisms exist for mass generation?

Section 1: Phenomenology

More: “What can the lattice say about certain models?”
Less: “How viable are these models?”

Compositeness

Strongly coupled BSM Models

- Besides the Higgs, the only scalar particles we know about in nature arise as bound states of a strongly interacting sector — QCD
  - Example: σ = $f_0(500)$, $f_0(980)$, etc…
  - The SM Higgs is a scalar. Maybe the SM Higgs comes from a new strong sector in the UV?
- From a Wilsonian perspective the Higgs masses is a relevant coupling. Does some symmetry protect it from large renormalization effects?
  - In QCD, pions are Goldstone bosons and are protected by shift symmetry
  - Maybe the Higgs is a (pseudo-) Goldstone boson?

Electroweak mass generation

In strongly coupled BSM models

Suppose a gauge-fermion sector in the UV confines and breaks chiral symmetry. If the chiral condensate…

… breaks $SU(2)_L$

"Technicolor"

- ~~No Higgs boson exists~~ ☠
- Higgs emerges from dynamics ("dilaton"?)
- Reasonable level of lattice investigation to date

… preserves $SU(2)_L$

"Composite Higgs"

- Higgs arises as an exact Goldstone boson from broken chiral symmetry
- Perturbative SM loops generate the Higgs potential and trigger EWSB
- Limited lattice investigation (!)

Today's focus

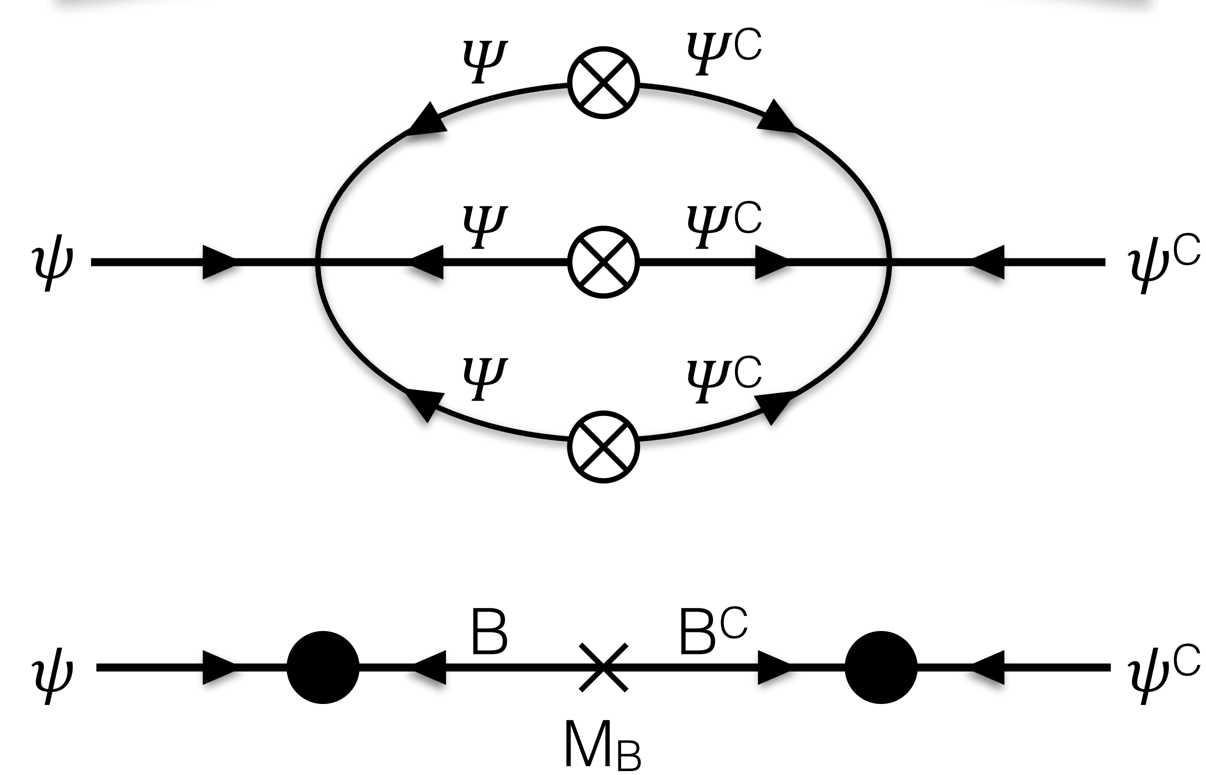
Fermion masses

via 4-fermion interactions

- Often, Standard Model fermions couple quadratically to UV operators in BSM models.
- Partial compositeness = linear coupling to baryon operators in the UV
    - Mass mixing yields top quark partners
    - Idea: Kaplan D.B., Nucl Phys B365 (1991) 259-278
- Realistic implementations are delicate in either case and must respect stringent constraints from flavor physics
    - See Panico, G. and Wulzer, A. “The Composite Nambu-Goldstone Higgs” (Springer 2016) for some references

$$\bar{\psi}\psi\bar{\Psi}\Psi \sim \bar{\psi}\psi\mathcal{O}_{\mathrm{ETC}}$$

$$\bar{\psi}\Psi\Psi\Psi \sim \bar{\psi}\mathcal{O}_{\mathrm{PC}}$$



The role of the lattice?

- Historically, phenomenology has mostly focused on IR descriptions via EFTs
  - EFTs are necessary for interpreting potential signals from LHC data
  - Results are expressed in terms of (undetermined) low-energy constants
  - Computations with UV degrees of freedom are hard by construction, since interactions are strong

➡ The lattice can compute LECs, masses, etc… But it needs a UV completion!

The role of the lattice?

➡ The lattice can compute LECs, masses, etc… But it needs a UV completion!

- Ferretti and Karateev (1312.5330) classified possible UV completions

A. Gauge group is anomaly-free

B. Gauge group contains the SM gauge group + custodial SU(2)

C. Theory is asymptotically free

} "Healthy" physical theory

D. Matter fields are fermionic irreps of the gauge group

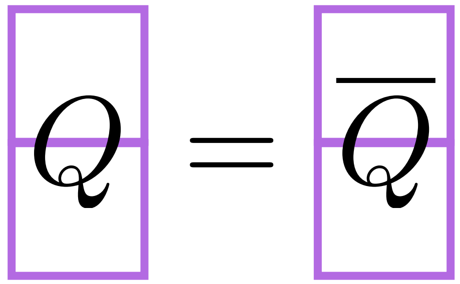
} (Sufficient?) Condition for partial compositeness

Ferretti's Model

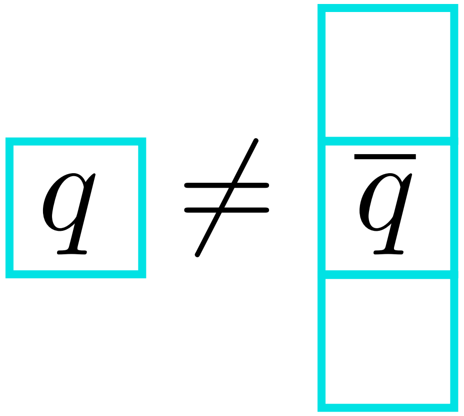
A "minimal" UV continuum theory of partial compositeness from (1404.7137)

- SU(4) gauge theory with "multirep" matter content
  - 5 two-index antisymmetric ("sextet") Majorana fermions
    - Equivalent DOF: "2.5 sextet Dirac fermions"
    - Sextet SU(4) is a real representation ≘ to SO(6)
  - 3 fundamental Dirac fermions
- Symmetry breaking: SU(5)/SO(5) in the IR (for sextets)
  - Symmetry breaking pattern *different from QCD*
- New territory for lattice simulations

real irrep



complex irrep



Ferretti's Model: FAQs

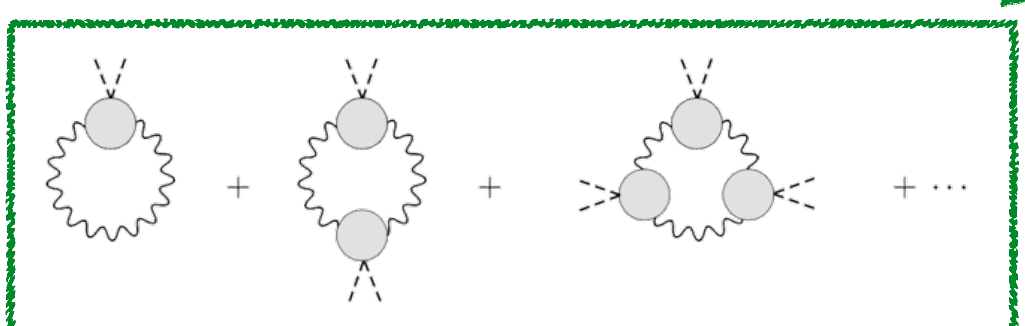
- Why SU(4) gauge theory?
  - ➡ Maintains asymptotic freedom for the desired fermion content
- Why sextet fermions?
  - ➡ Higgs $\in$ SU(5)/SO(5) works, is reasonably minimal, and has been studied via EFT in the IR. Sextet fermions produce this pattern of symmetry breaking.
- Why the particular global symmetry structure?
  - ➡ The IR theory must contain the Standard Model + custodial SU(2) after SSB $G_F \rightarrow H_F$ in the UV

Ferretti's Model

EWSB via top-driven vacuum misalignment

- χSB occurs in UV, where the future Higgs begins life as an exact Goldstone boson.
- Then include perturbative interactions with the Standard Model:
  - EW gauge bosons induce a positive potential via the mechanism of "vacuum alignment."
    - The physics is identical to EM mass splittings between pions in QCD.
    - These interactions do *not* trigger EWSB.
  - The top quark induces a negative potential. If this effect is large enough, "vacuum misalignment" drives the formation of a Higgs VEV and triggers EWSB.

$$V_{\text{eff}}(h) \sim (\alpha - \beta)\left(\frac{h}{f}\right)^2 + \mathcal{O}(h^4)$$



Low-energy constants,
Calculable on the lattice

Our lattice deformation

(What we actually simulate)

- Still SU(4) gauge theory, but modified matter content
  - 2.5 $\longmapsto$ 2 Dirac sextet SU(4) fermions
  - 3 $\longmapsto$ 2 Dirac fundamental SU(4) fermions
- Symmetry breaking: SU(4)/SO(4) in the IR (for sextets)
  - Still a rich system for lattice investigation
  - Expected to capture the important qualitative features of Ferretti's model

Technical specifications

- Multirep MILC code - Y. Shamir
- Lattice discretization
    - NDS gauge action - T. Degrand, Y. Shamir, and B. Svetitsky (1407.4201)
    - nHYP smearing
    - Clover-improved Wilson fermions, clover coefficient c_{SW} set to unity
- Gauge generation with hybrid Monte Carlo algorithm
- Today in this talk:
    - First-ever simulations with simultaneous dynamical fermions in multiple representations (in 3+1 dim)
    - Preliminary “zero-temperature” meson spectroscopy across dozens of ensembles
    - Highlights from some preliminary work toward the Higgs potential and baryons

Scale setting: the Wilson Flow

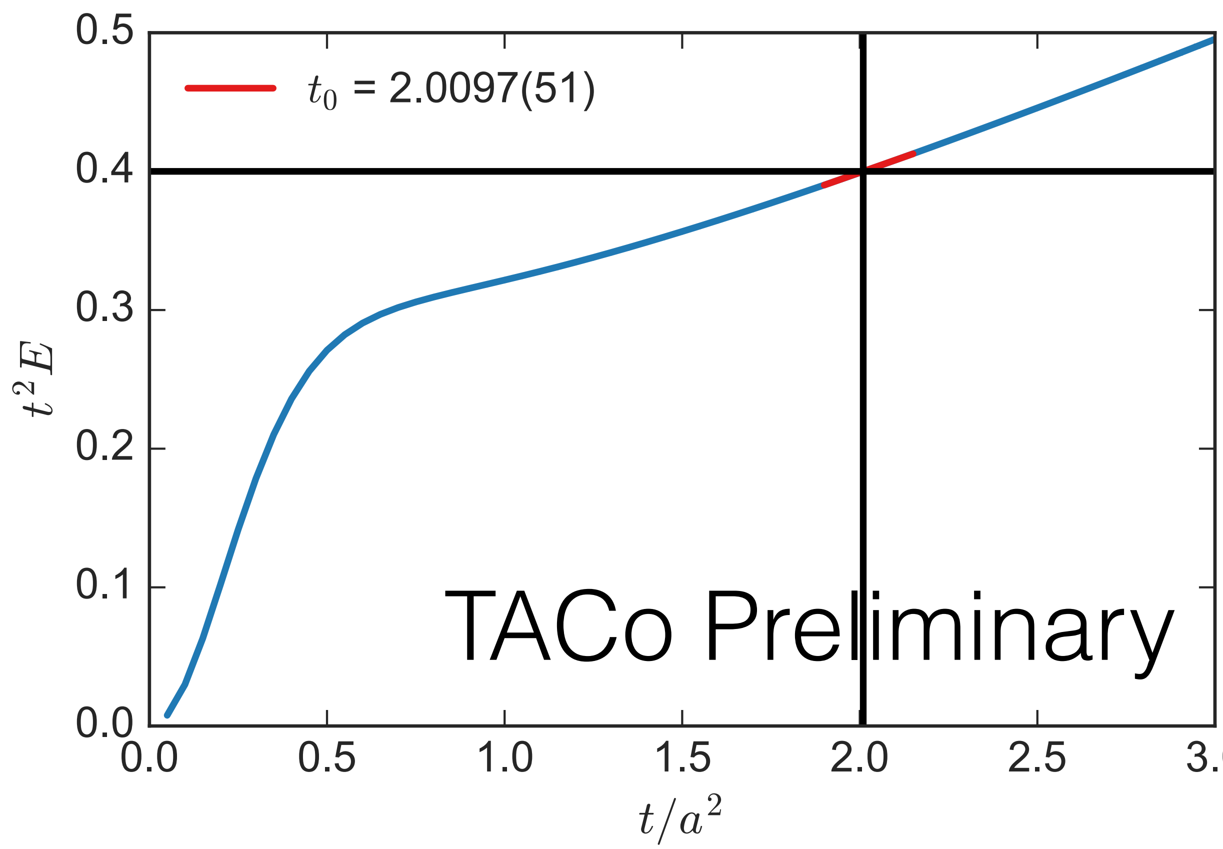
“Always look at dimensionless ratios”

- We set the scale with the Wilson flow scale, t_0
- Flow the gauge fields in a fictitious 5th dimension (e.g., Lüscher: 1006.4518)
- Consider observables built from the flowed field strength to define a reference scale

$$\langle E(t) \rangle \sim \langle G^2(t) \rangle$$

$$t_0^2 \langle E(t_0) \rangle \stackrel{!}{=} M(N_c)$$

- In QCD, $\sqrt{t0}$ = 0.14 fm with $M(N_c{=}3) = 0.3$
- Large-N: $t_0 \sim N_c$, so take $M(N_c{=}4) = 0.4$
- DeGrand (1701.00793) gives details, compares to other scale setting schemes, and provides more careful connection to large-N



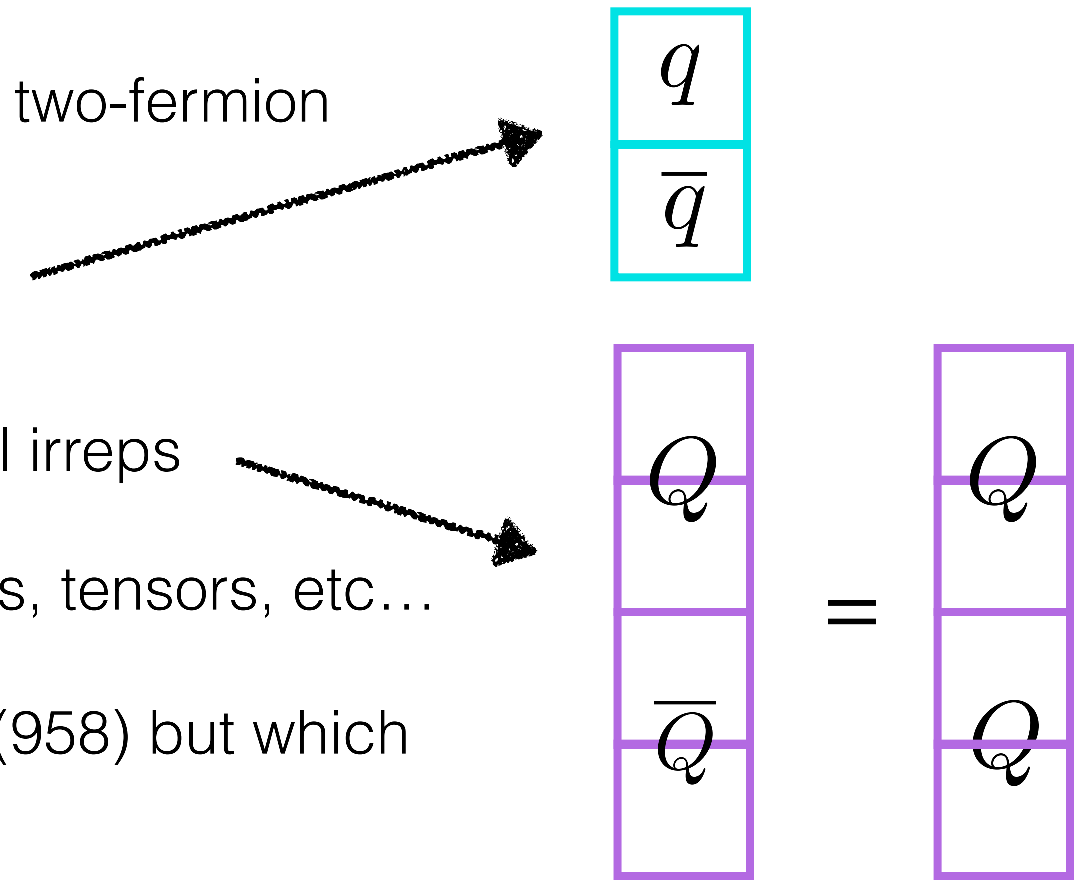
$$\boxed{(m\cdot a) \times (\sqrt{t_0}/a) = \text{dimensionless}}$$

Section 2:
Meson spectroscopy

"Study the low energy degrees of freedom first."

Mesons in Multirep SU(4)

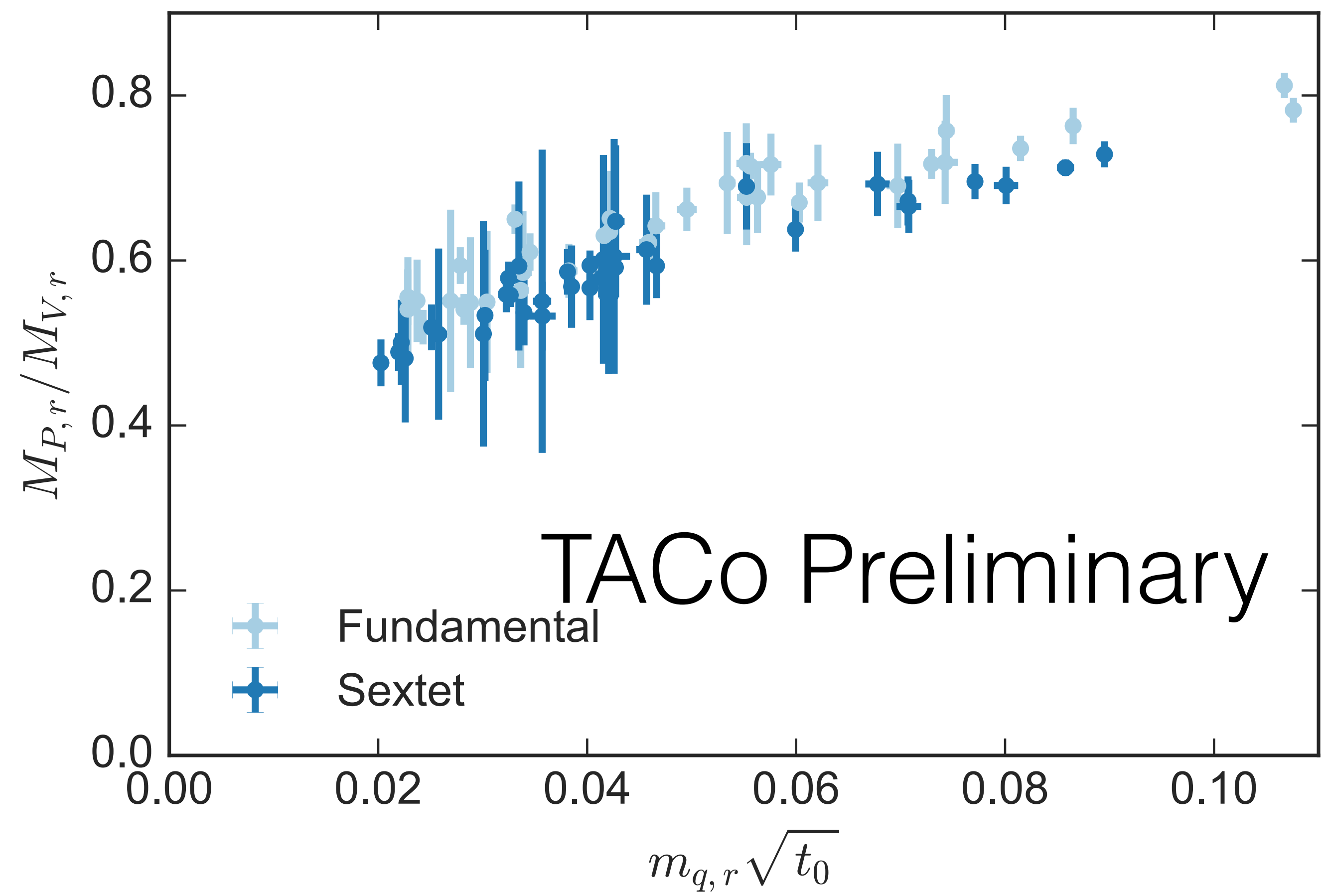
- The meson-like spectrum consists of color-singlet two-fermion objects and is mostly like QCD
  - Fundamental mesons — analogous to QCD
  - Sextet mesons — “mesons = diquarks” in real irreps
  - Scalars, pseudosclars, vectors, pseudovectors, tensors, etc…
- Key difference — flavor-singlet analogue of the η’(958) but which is an exact Goldstone boson
  - Superposition of fundamental, sextet, and “glue”
  - Tricky to measure directly on the lattice…
  - Nothing more here, but an interesting feature of the model…



Overview of Ensembles

Pseudoscalar-to-vector mass ratio: M_P/M_V

- O(40) total ensembles
- Volumes: $16^3 \times 32$ $\quad 16^3 \times 18$
- Fermion masses m_q from the axial Ward identity
- Meson masses from 2-point functions
- $0.5 \lesssim M_P/M_V \lesssim 0.8$
  - QCD language: "$M_P \gtrsim 450$ MeV"
- Comparable behavior in both fermion representations



$$M_\pi/M_\rho = 0.18$$
$$M_{K^\pm}/M_{K^*} = 0.55$$
$$M_{\eta_c}/M_{J/\psi} = 0.96$$
$$M_{\eta_b}/M_\Upsilon = 0.99$$

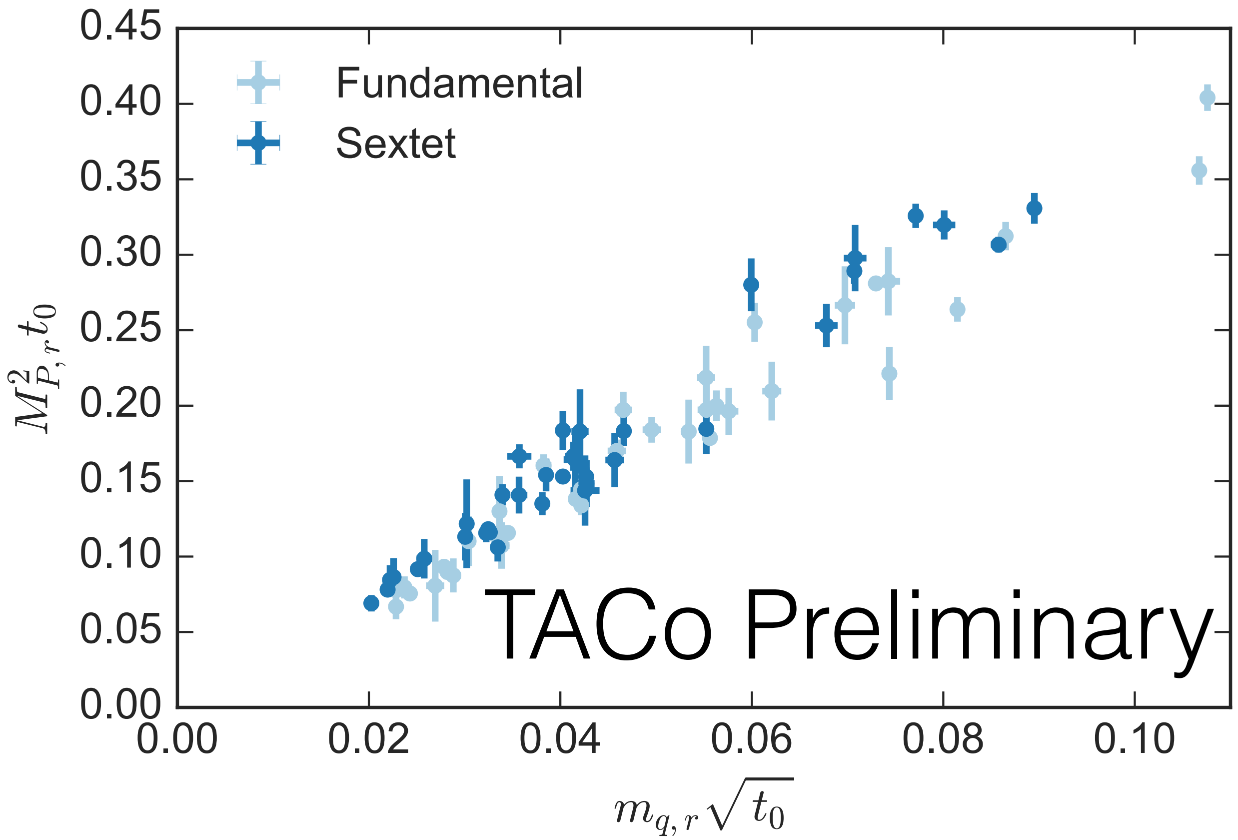
Pseudoscalar masses

- Cancel lattice spacing with dimensionless “ratios” using t_0
- Leading-order ChiPT says:

$$M_P^{\ 2} \sim m$$

- (Plausibly) linear behavior
- Lattice artifacts?
    - (Clover-improved) Wilson fermions are not chiral
    - Some remnant additive renormalization?

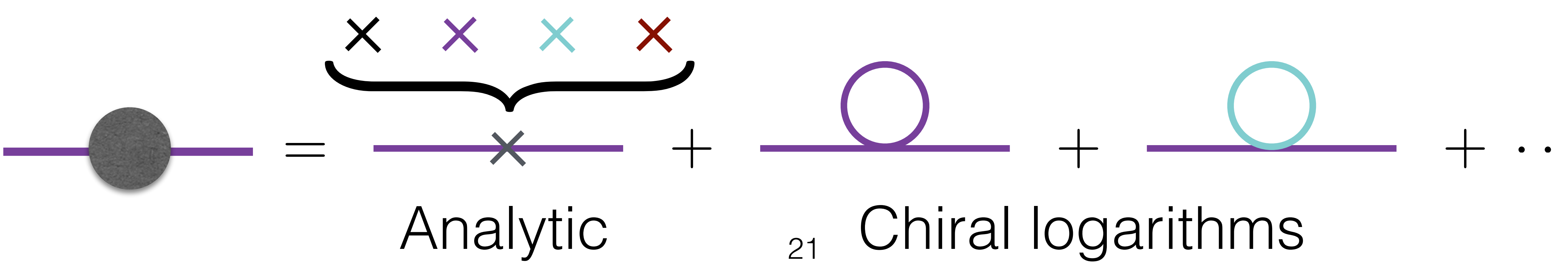
➡ Try to model data with EFT: ChiPT



Goldstone bosons and EFT

(A 5-minute review of ChiPT at NLO)

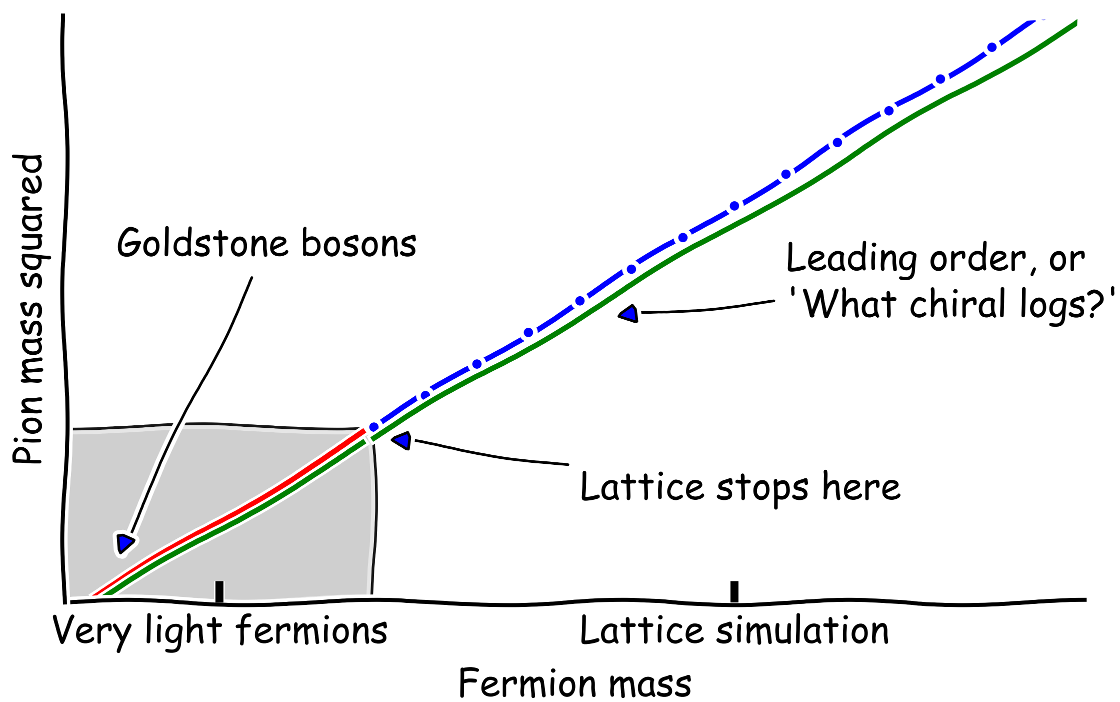
- Multirep ChiPT worked out to NLO by DeGrand, Golterman, Neil, and Shamir in 1605.07738
    - Schematically similar to single-rep ChiPT with analytic terms and chiral logarithms
- Wilson ChiPT at NLO suggests ($M_P^2 t_0$) and ($F_P\sqrt{t_0}$) also depend explicitly on the lattice spacing through (ma) and ($a/\sqrt{t_0}$)
    - Work within some power-counting scheme, e.g., "$p^2 \sim a \sim m$"
    - Coefficients involving the lattice spacing are lattice artifacts



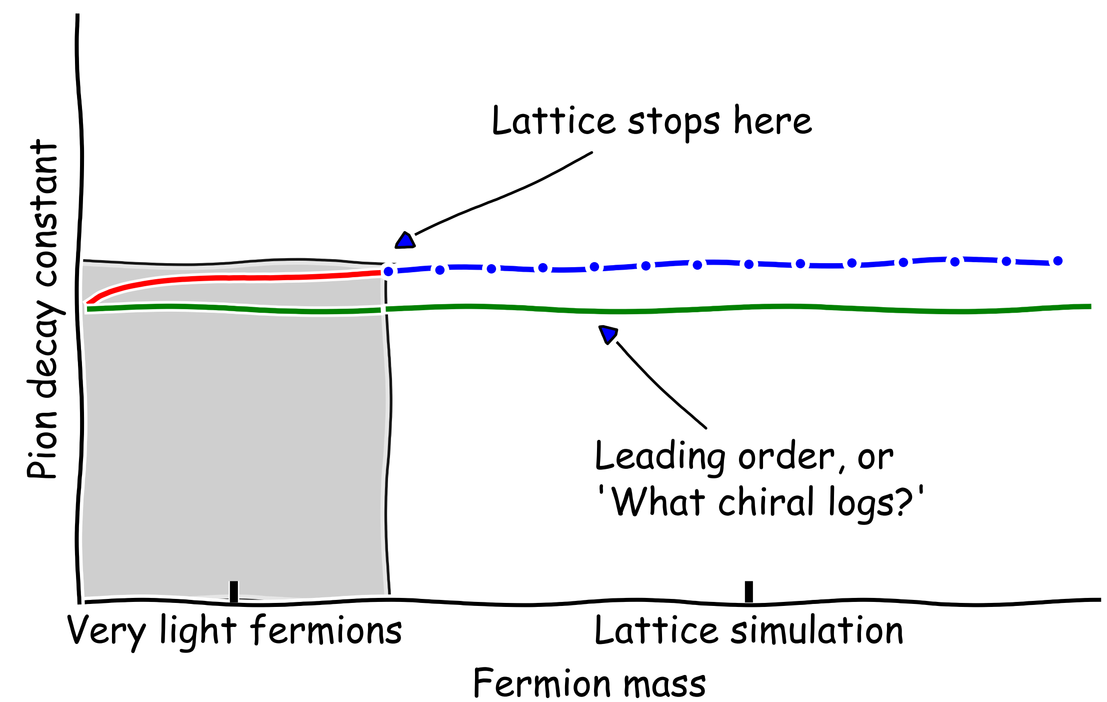
Goldstone bosons and EFT

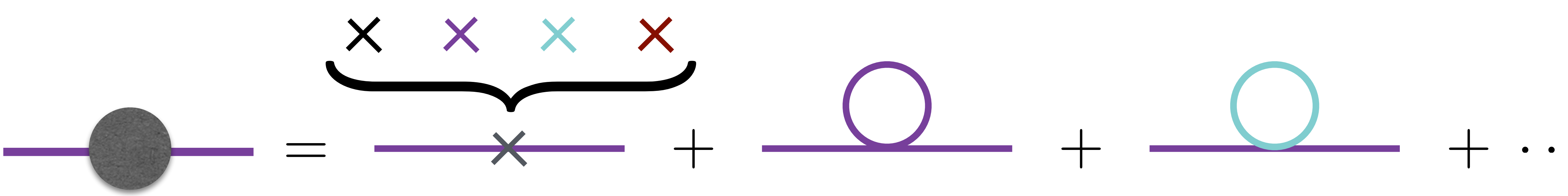
(A 5-minute review of ChiPT at NLO)

M_π^2 vs m_q



F_π vs m_q



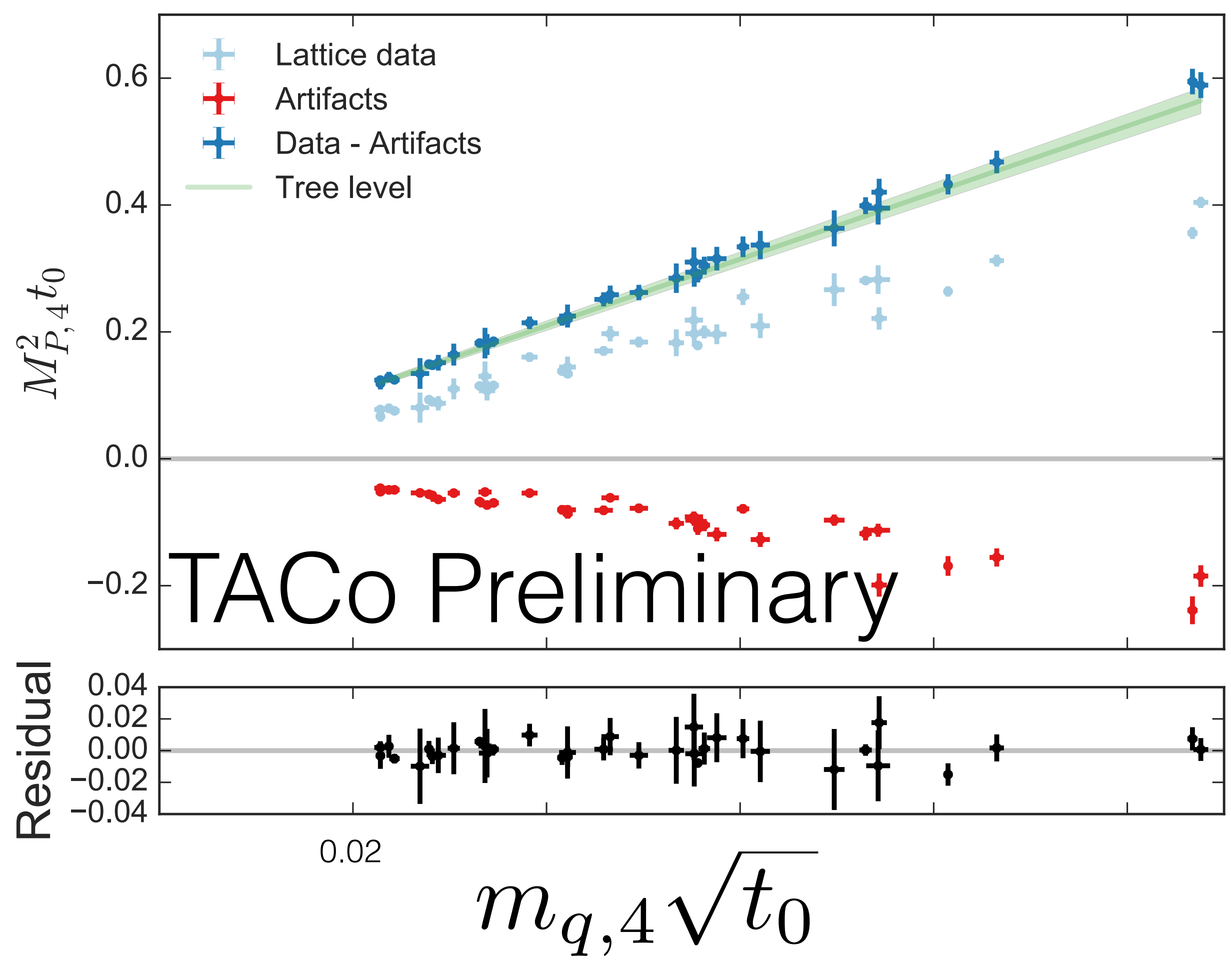


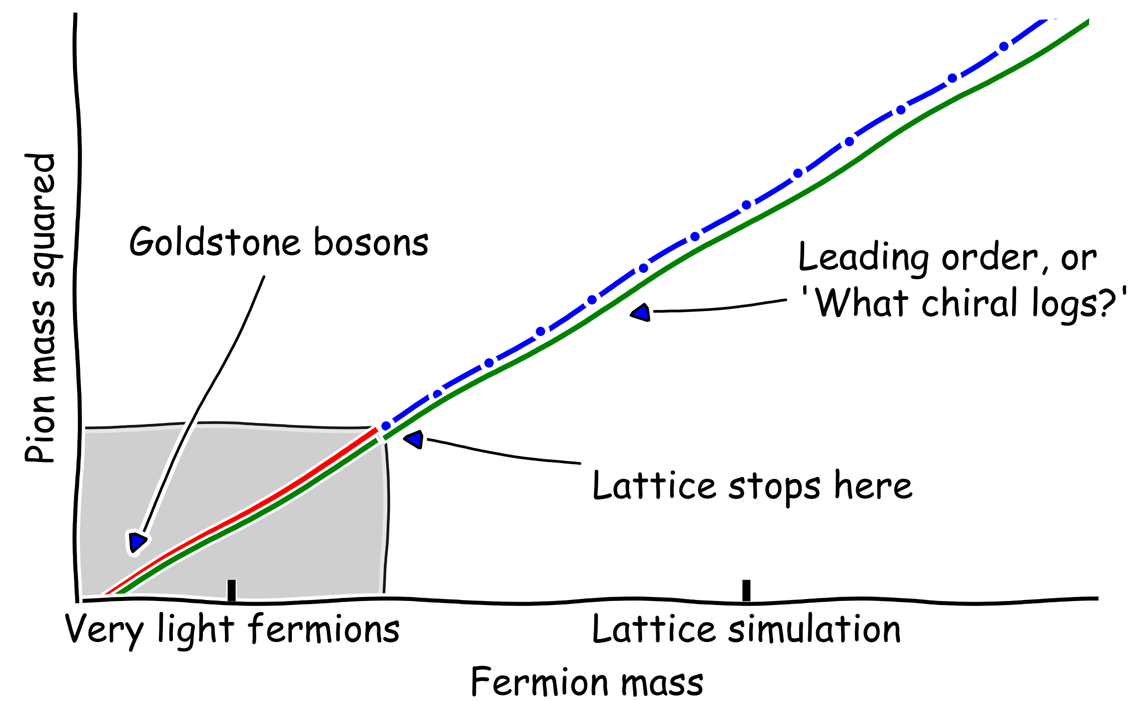
Analytic

Chiral logarithms

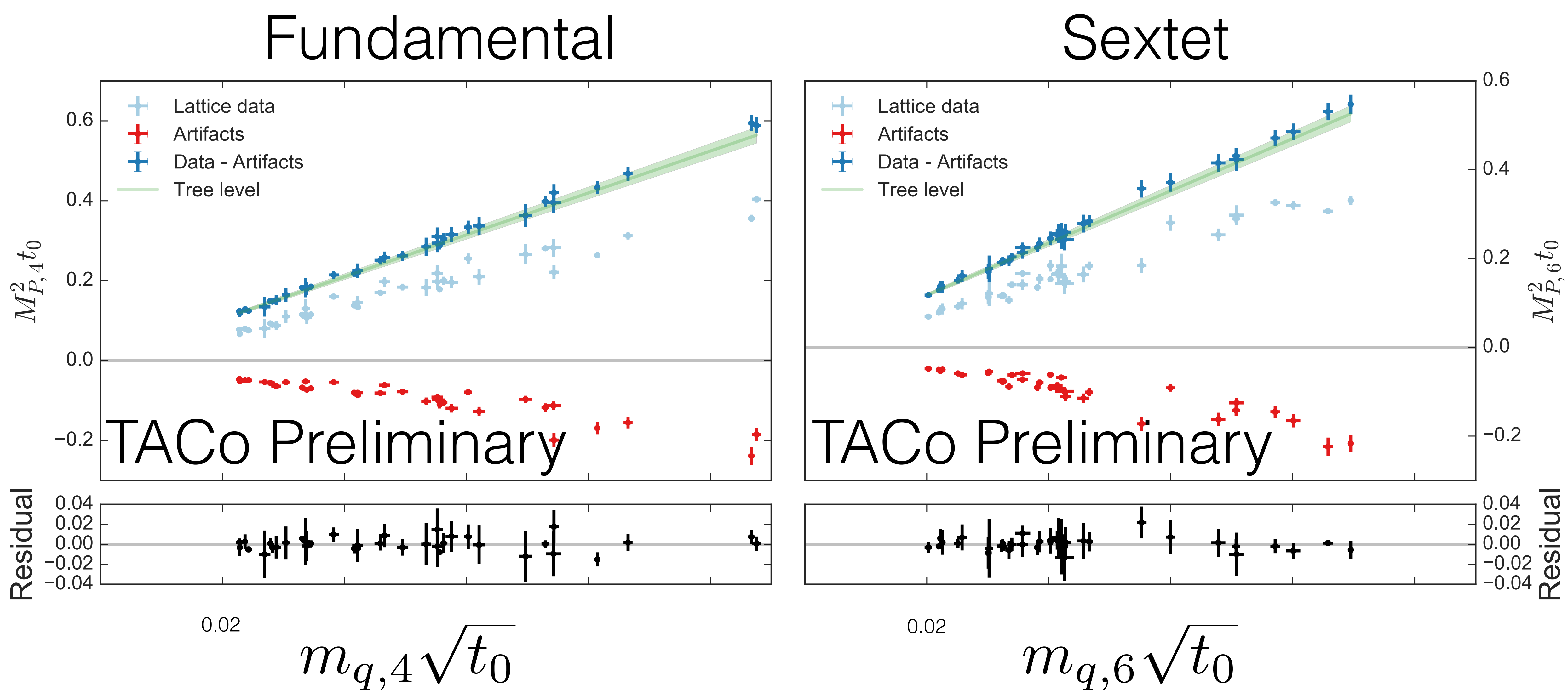
Goldstone bosons on the lattice: M_P^2

Fundamental

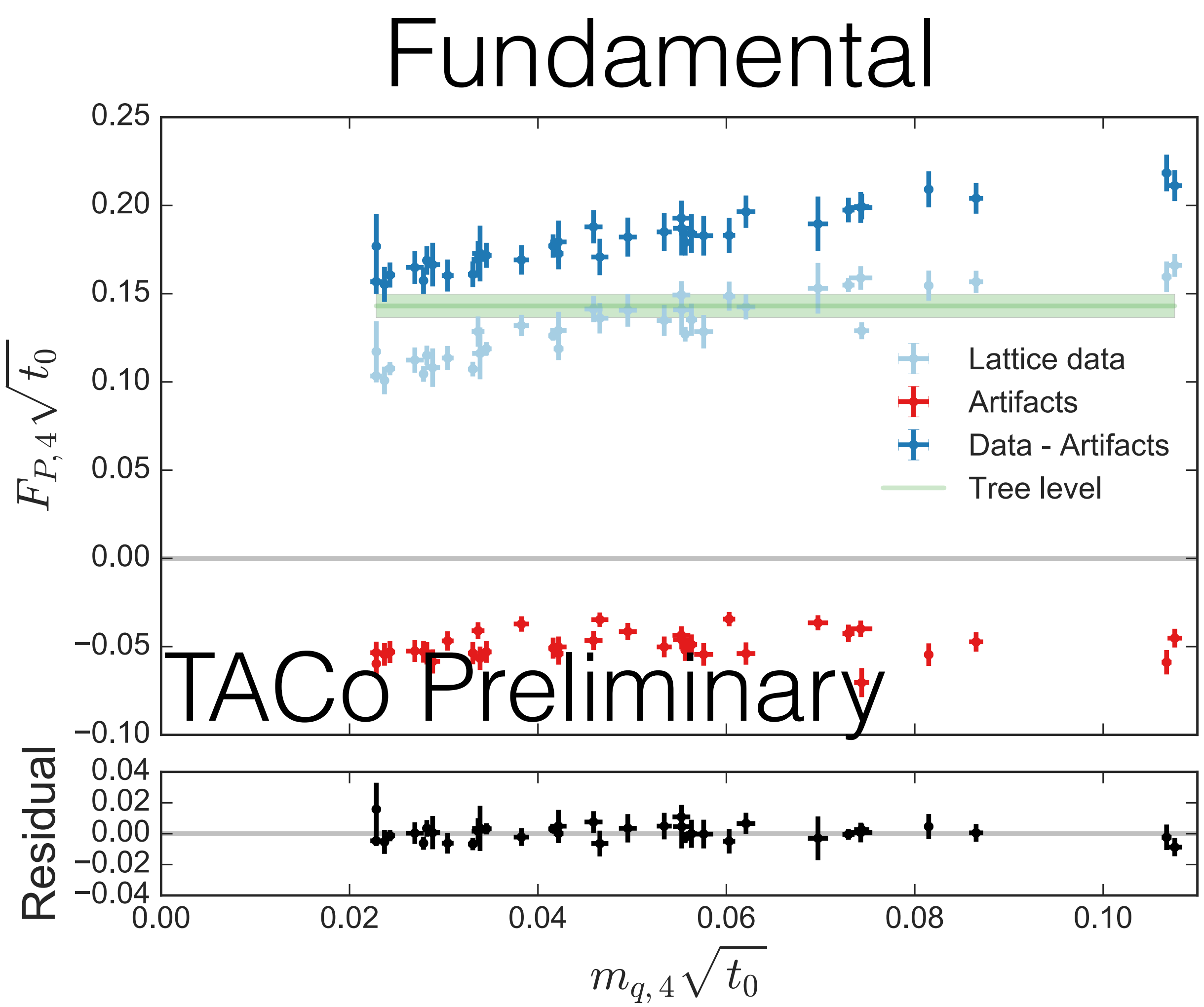


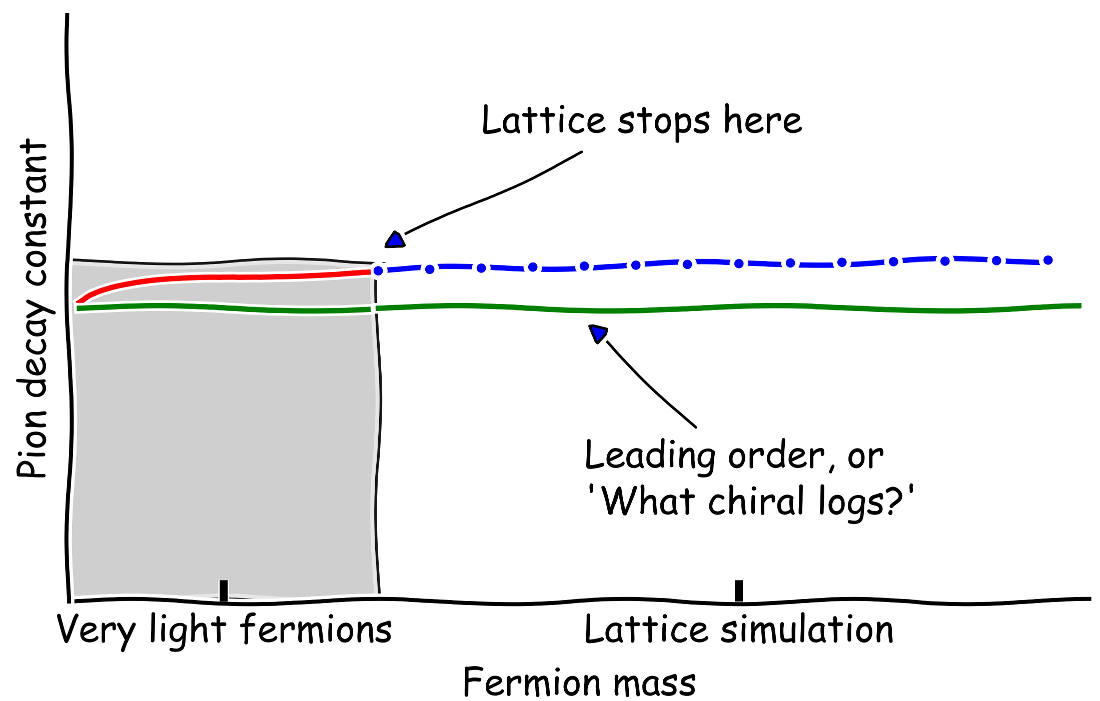


Goldstone bosons on the lattice: M_P^2

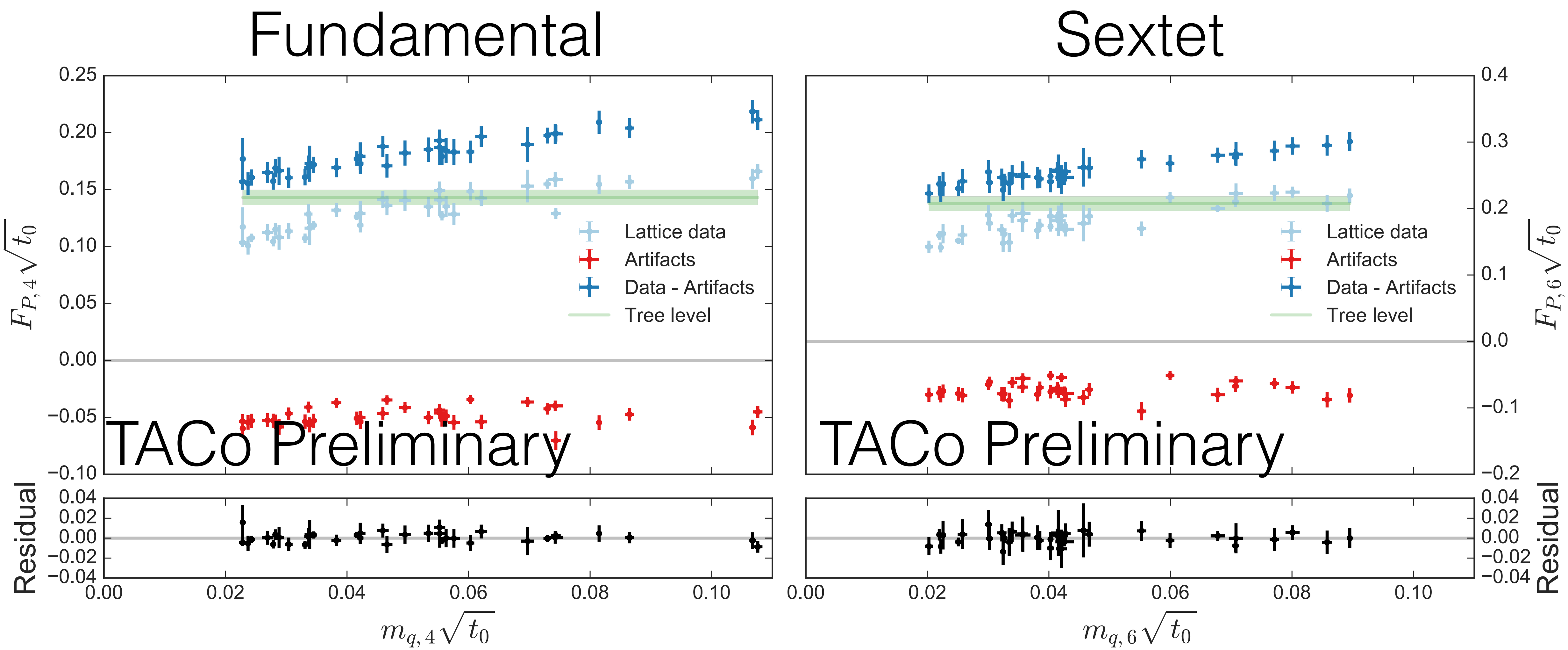


Goldstone bosons on the lattice: F_P

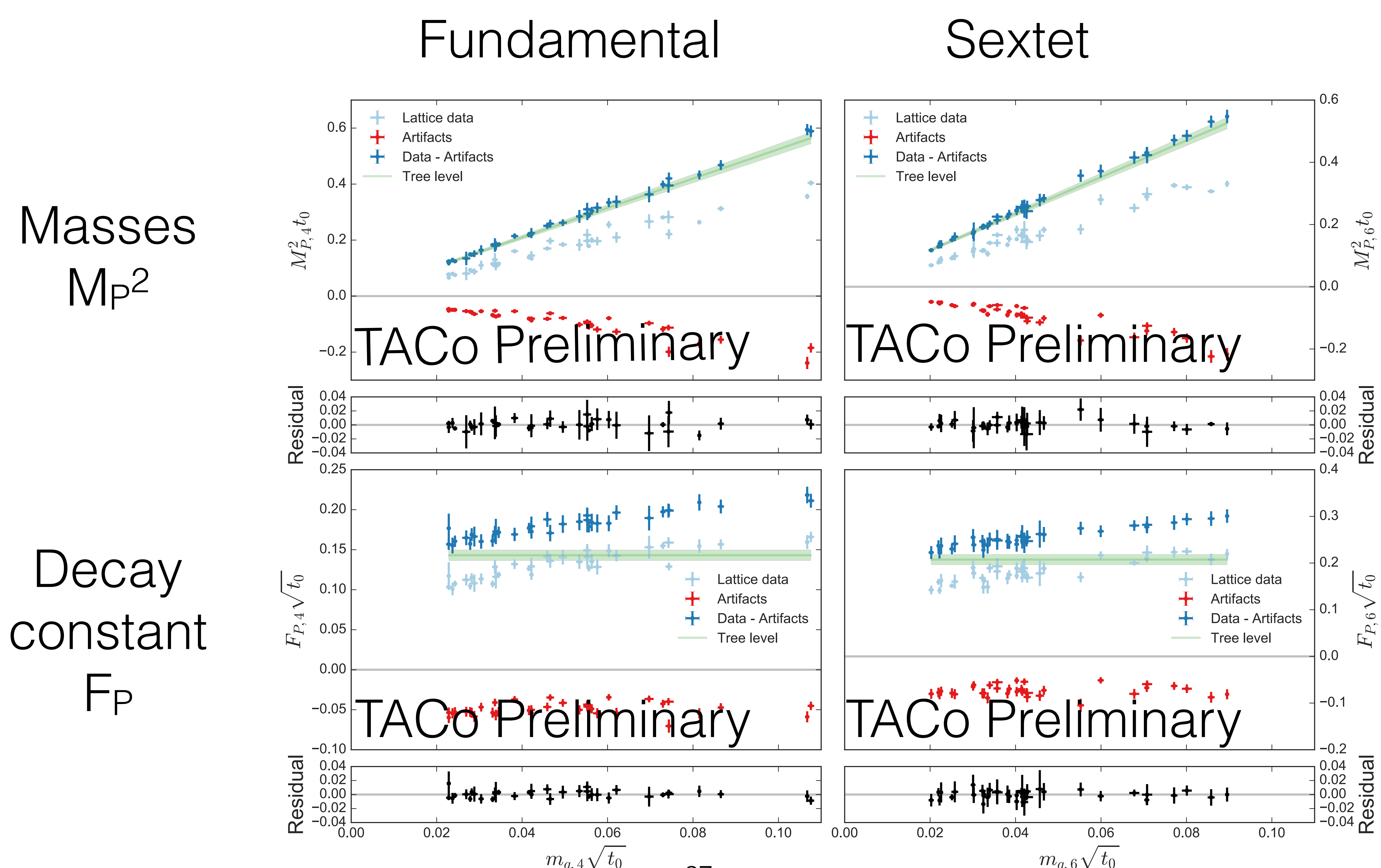




Goldstone bosons on the lattice: F_P



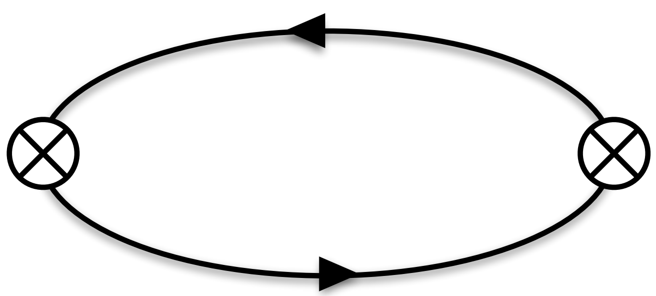
The Goldstone bosons on the lattice



Scaling relations and “large-N”

An interlude

Usual large-N story for fundamental fermions



$$\sim \langle JJ \rangle \sim a_n^2 \sim \boxed{N} \sim \boxed{\dim(r)}$$

$$\Longrightarrow \quad \langle 0|J|\text{meson}\rangle \sim \boxed{\sqrt{N}} \sim \boxed{\sqrt{\dim(r)}} \sim F_{\text{meson}}$$

Suggestive (but heuristic) generalization to other reps

Irrep	dim(r)	N≫1	N=4
F	N	N	4
AS_2	$(N^2-1)/2$	$N^2/2$	8

➡ The lattice tests these rough arguments non-perturbatively

Condensates

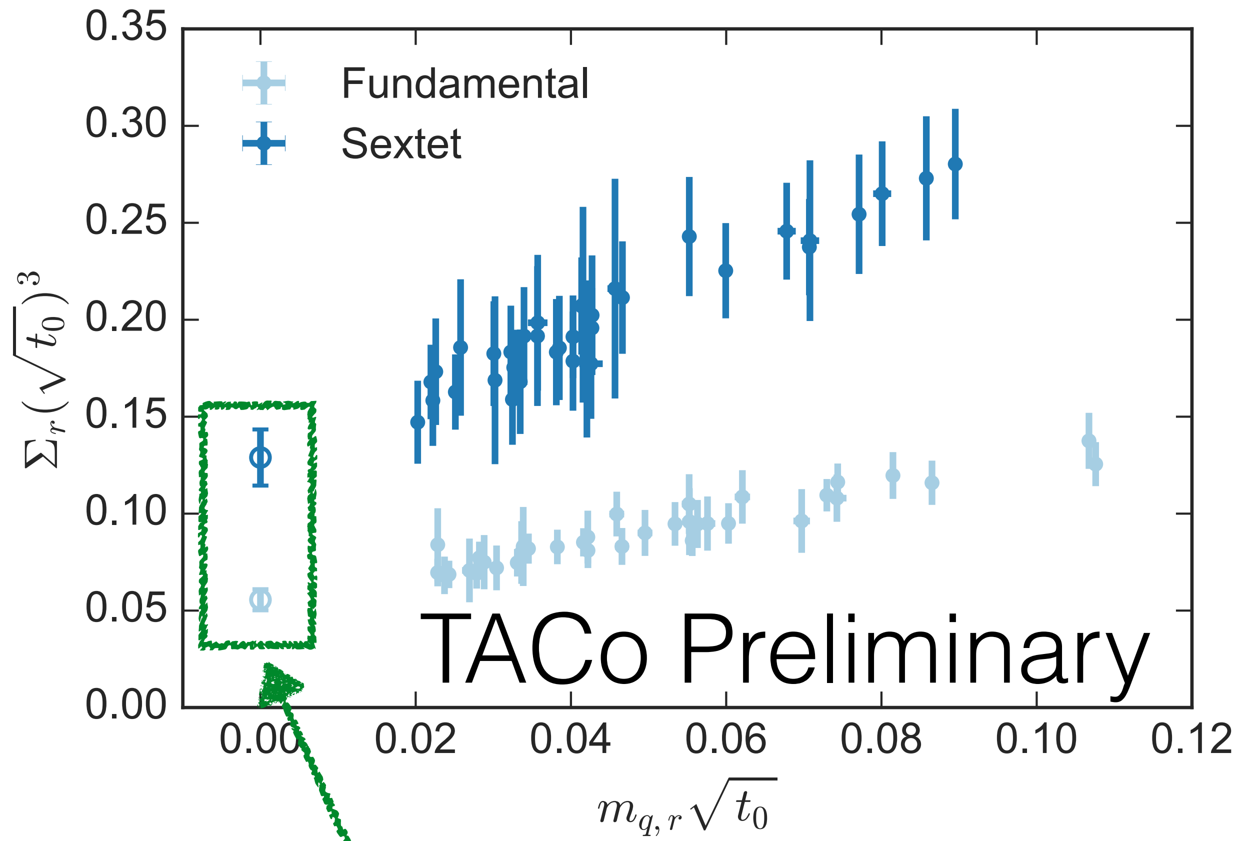
- Direct measurement of chiral condensates is tricky
- But ChiPT predicts them!

$$\Sigma_r = \frac{M_{P,r}^2 F_{P,r}^2}{2m_{q,r}} \xrightarrow{m_q \to 0} B_r F_r^2$$

Measured quantities

Passage to chiral limit

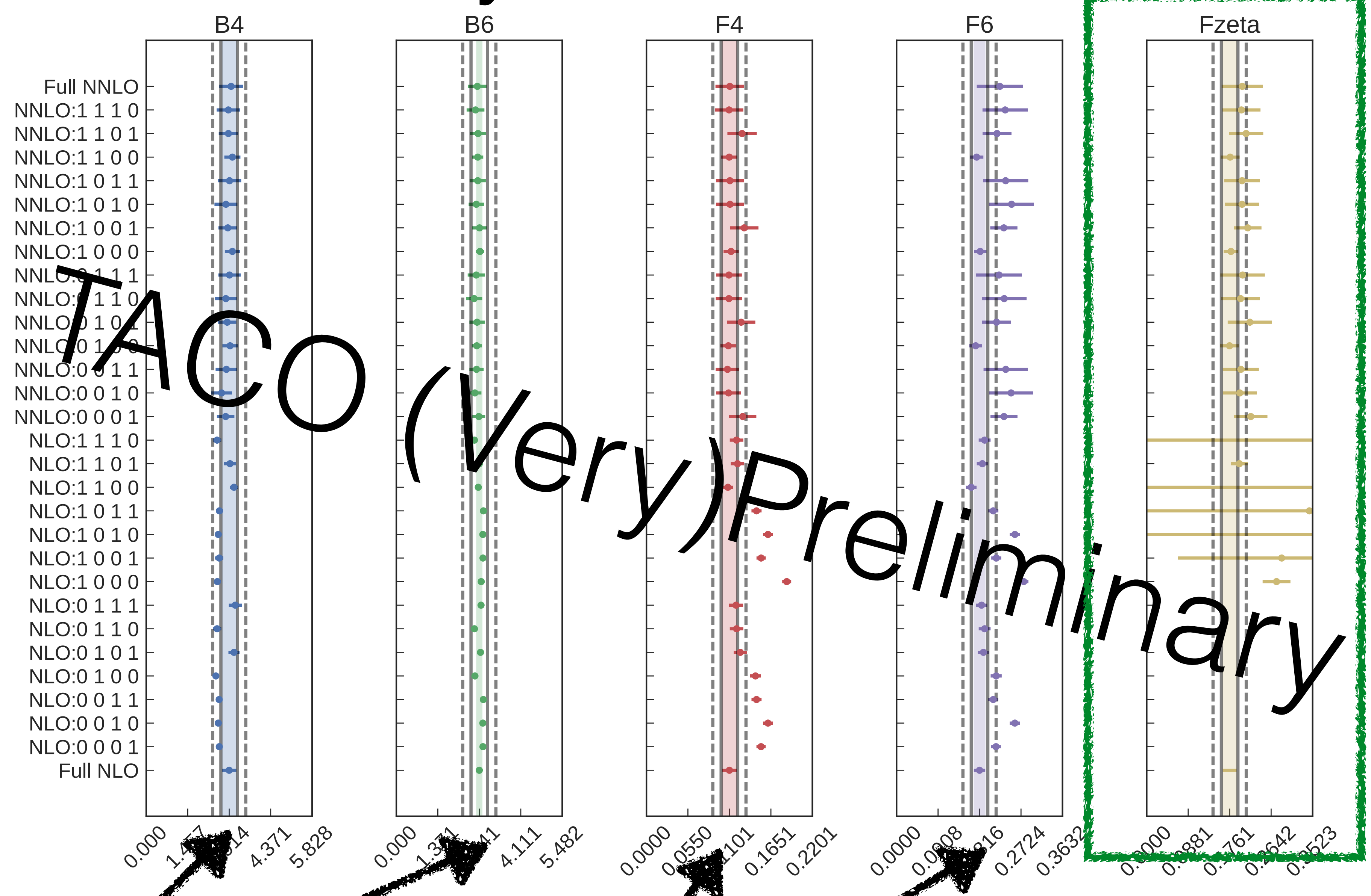
LECs, from ChiPT analysis



$$\frac{\Sigma_6}{\Sigma_4} \sim \frac{\dim(\mathrm{AS}_2)}{\dim(\mathrm{Fund})} \overset{N \gg 1}{\Longrightarrow} \frac{N^2/2}{N} = \frac{N}{2} \overset{N=4}{\Longrightarrow} 2$$

And hot off the press…

1:51 PM, 27 July 2017



“Slope” of M_P^2

Decay constants

From a novel “multirep” chiral log

Vector mesons

- Measure masses and decay constants, just as for the pseudoscalar
- Use (ChiPT-inspired) empirical functions to model measurements, estimate lattice artifacts
- Interesting for phenomenology, since vector resonances are often the target of collider searches
- Vector information is most interesting when combined with data from the Goldstone sector

Decay constants

F_V/F_P in a fixed representation

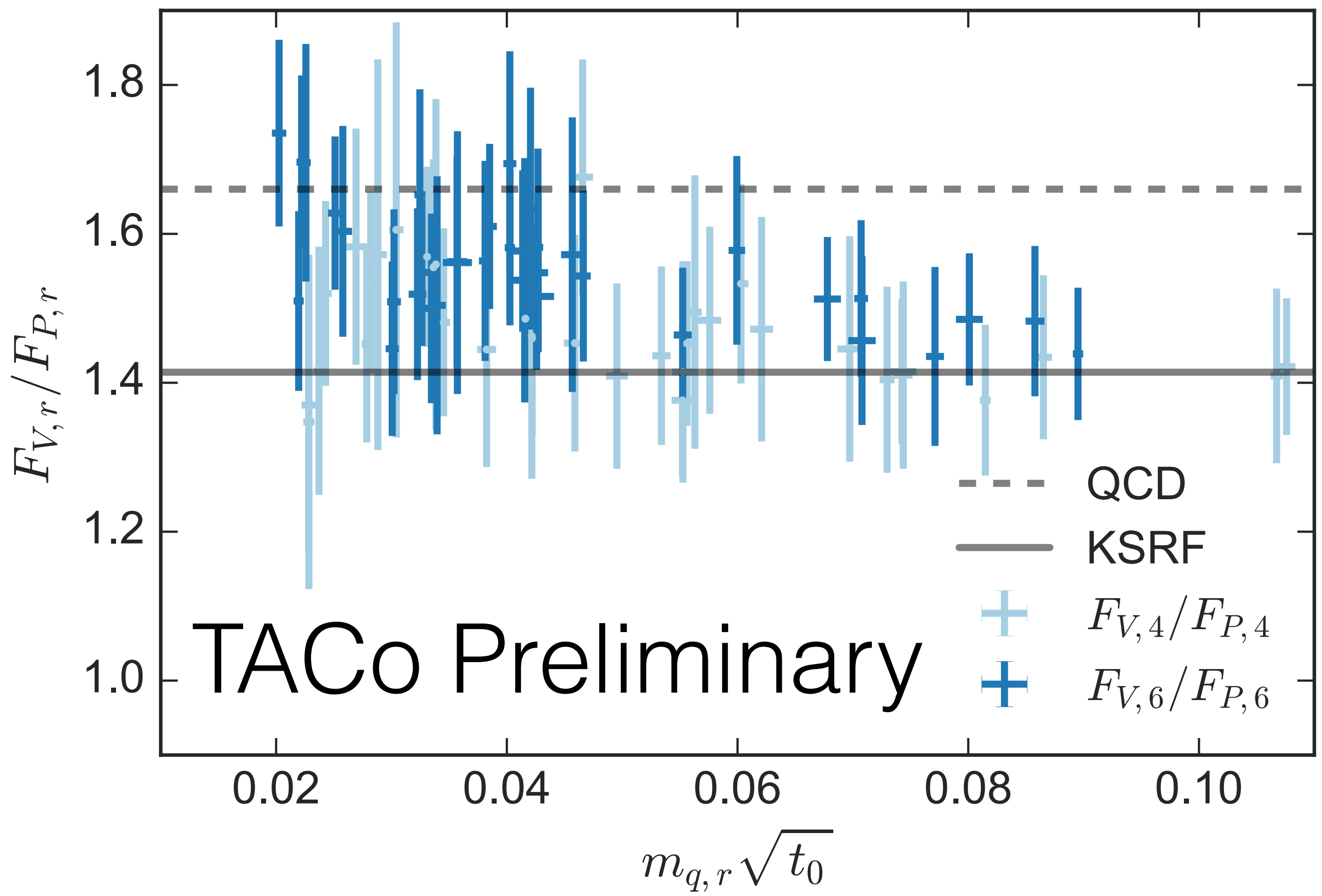
- A priori, F_V and F_P are unrelated
- KSRF (1966) related F_V and F_P using current algebra and vector meson dominance:

$$F_V = \sqrt{2} F_P$$

- Vector meson dominance is an uncontrolled but enlightening and physically motivated approximation
- QCD experiment:

$F_V/F_P \sim 216$ MeV/130 MeV ~ 1.66

- Success is comparable to that of QCD
- Both representations are comparable



✦ K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16, 255 (1966)
✦ Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966)
✦ KSRF away from QCD in a different BSM model: 1601.04027

Decay widths via KSRF

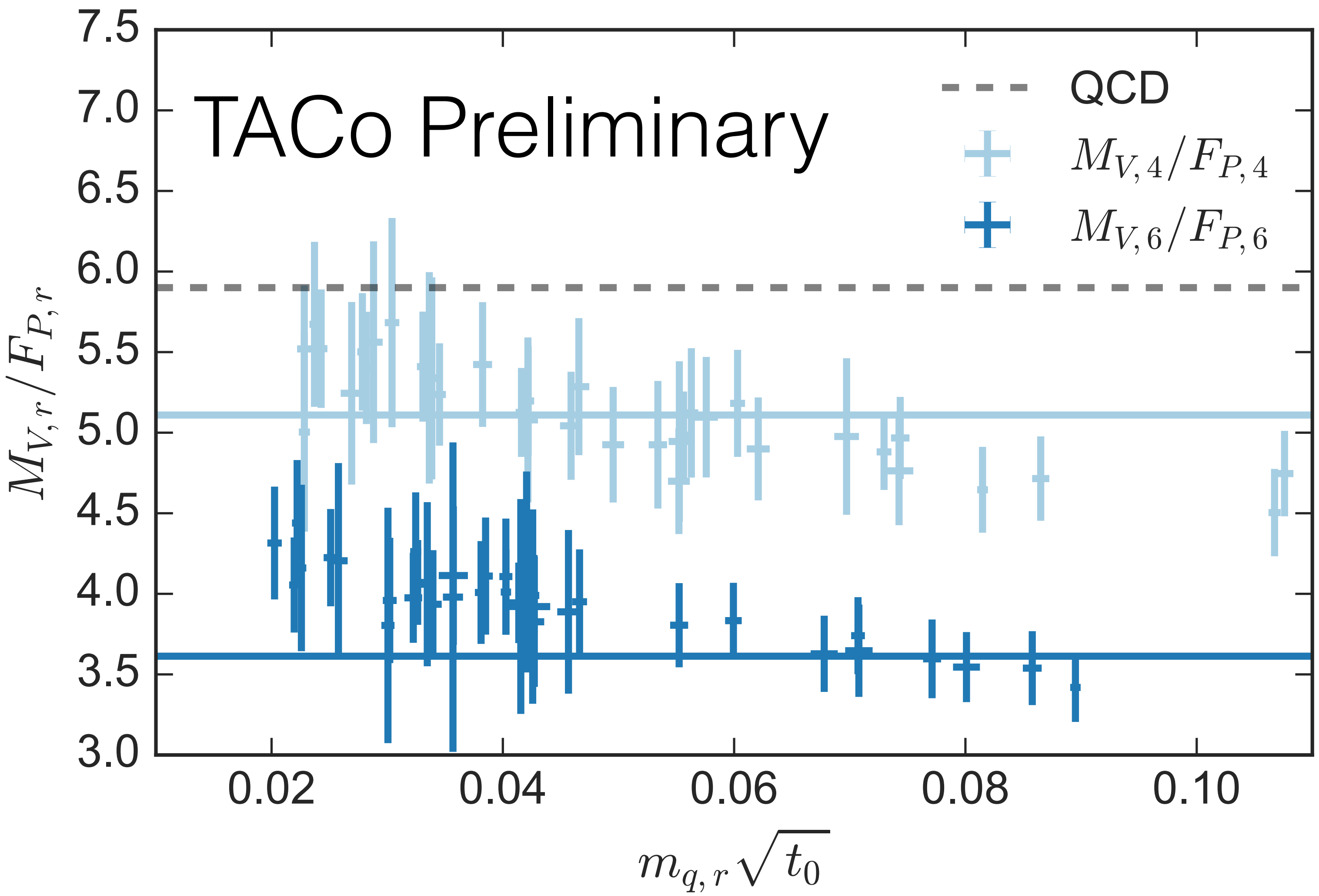
- KSRF (1966) also predict the coupling strength:

$$g_{VPP} = \frac{M_V}{F_P}$$

- This coupling allows for tree-level estimation of the vector width:

$$\Gamma_V \simeq \frac{g_{VPP}^2 M_V}{48\pi}$$

$\left\{ \begin{array}{c} \text{Polarization average} \\ \text{+ Phase space} \end{array} \right\}$

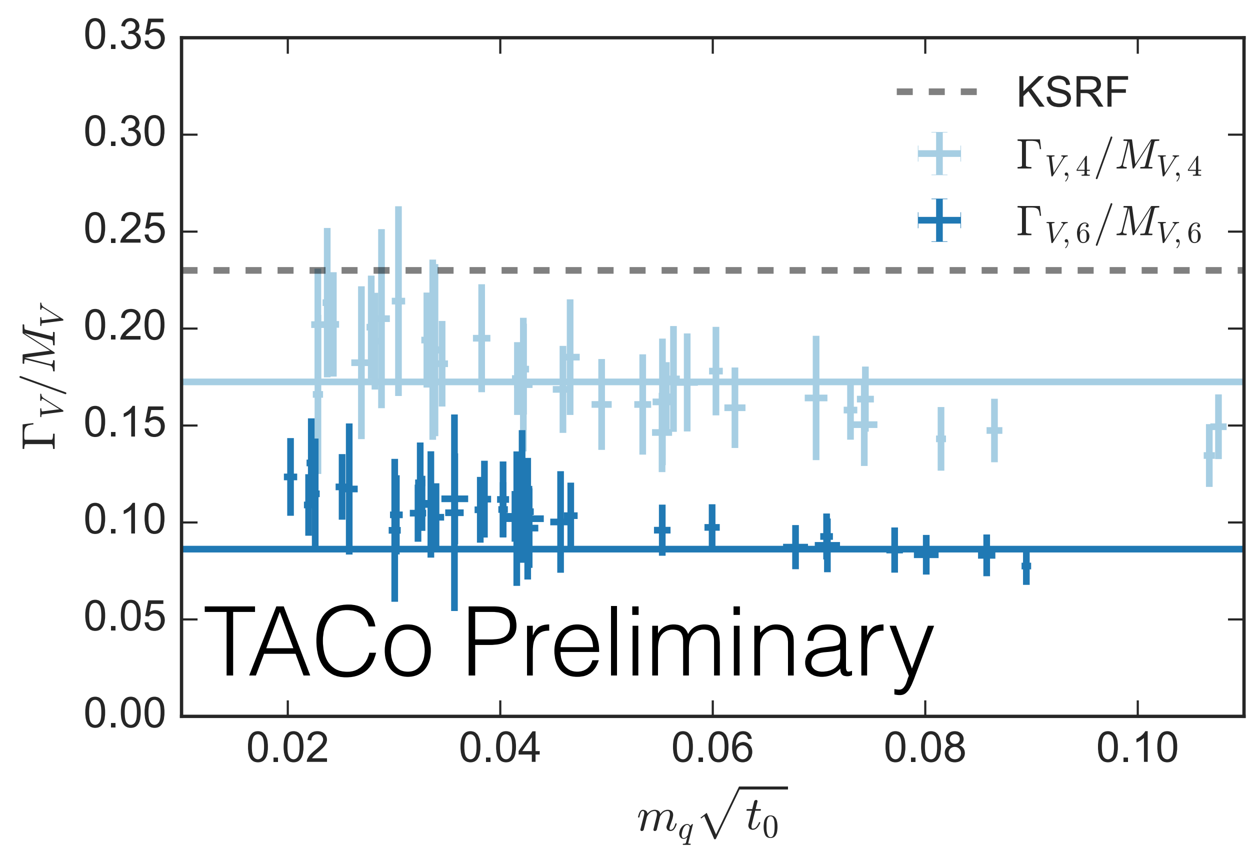


Decay widths via KSRF

- KSRF prediction:

$$\frac{\Gamma_V}{M_V} \simeq \frac{M_V^2}{48\pi F_P^2}$$

- Broad states, although likely narrower than $\rho(770)$ in QCD
    - Γ_{V6}/M_{V6} ~0.1
    - Γ_{V4}/M_{V4} ~0.2
- Assumes $M_P \ll M_V$, a good approximation is BSM models where P is the Higgs.



Section 3:
The Higgs potential

Mostly highlights from pilot lattice study 1606.02695

The Higgs Potential

- The Higgs begins life as an exact Goldstone boson from broken chiral symmetry in the UV
- EW gauge bosons induce a positive potential via the mechanism of "vacuum alignment."*
  - ✦ The physics is identical to EM mass splittings between pions in QCD.
  - ✦ These interactions do *not* trigger EWSB.

Compute this LEC on the lattice

$$V_{\text{eff}}(h) \sim (\boxed{(\alpha} - \beta)\left(\frac{h}{f}\right)^2 + \mathcal{O}(h^4)$$

$$\overbrace{m_{\pi^\pm}^2 - m_{\pi^0}^2} \sim \underbrace{\frac{\alpha_{\text{EM}}}{4\pi}\Lambda_{\text{QCD}}^2}_{\text{Dimensional analysis}} \sim \frac{\alpha_{\text{EM}}}{f_\pi^2}\int_0^\infty \underbrace{dQ^2 \Pi_{\text{LR}}(Q^2)}_{\text{Careful computation in field theory, Das (1967)}}$$

*That α>0 is *proven*: E. Witten, "Some Inequalities Among Hadron Masses," PRL 51, 2351 (1983)

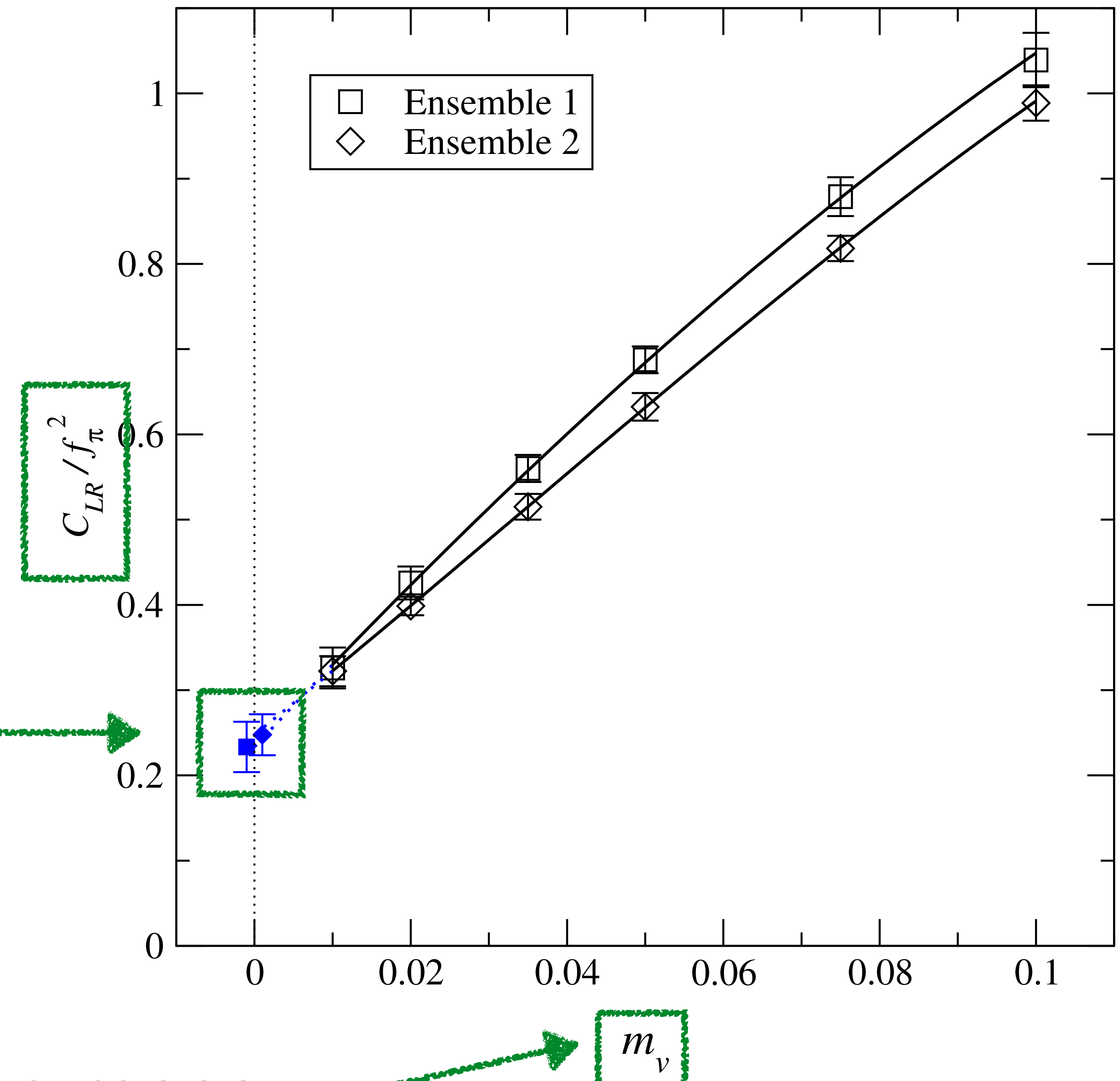
QCD version: Das *et al* (1967), Phys. Rev. Lett., 18, 759–761

Lattice Pilot Study

SU(4) single-rep simulation (1606.02695)

The low-energy constant

Extrapolate to the chiral limit

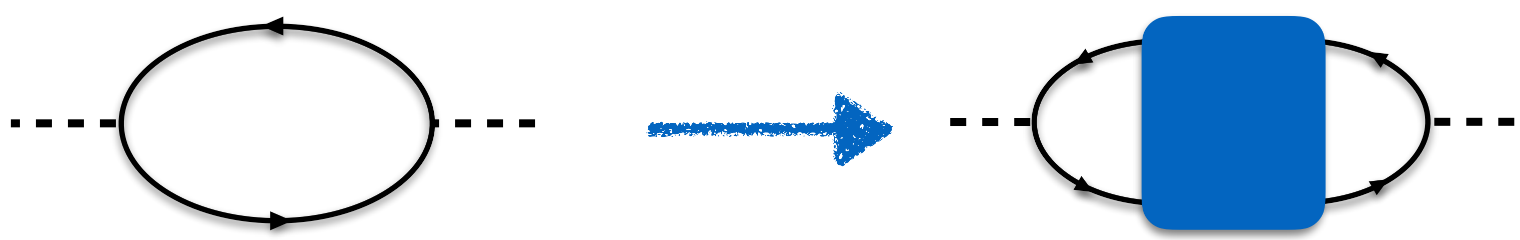


(Valence) fermion mass

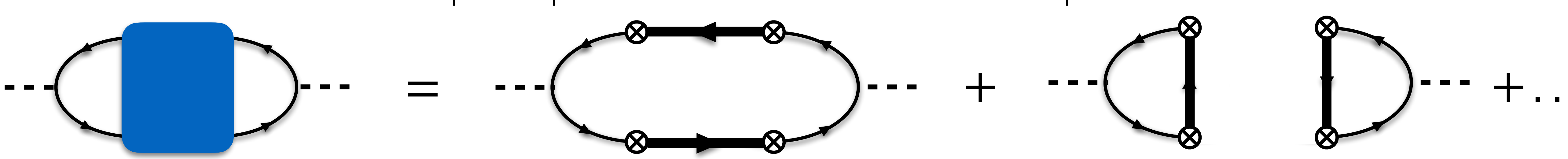
The Higgs Potential

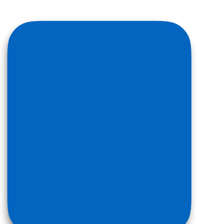
- The top quark induces a <u>negative</u> potential. If this effect is large enough, "vacuum misalignment" drives the formation of a Higgs VEV and triggers EWSB.

$$V_{\text{eff}}(h) \sim (\alpha - \boxed{\beta}) \left(\frac{h}{f}\right)^2 + \mathcal{O}(h^4)$$



SM Top Loop — Partial Compositeness





= lattice task = baryon 4-pt function

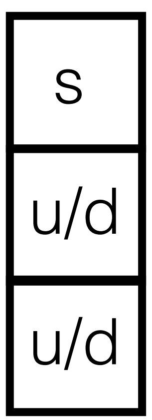
- Technically challenging, see 1502.00390 and 1707.06033
- Factorization at large-N?

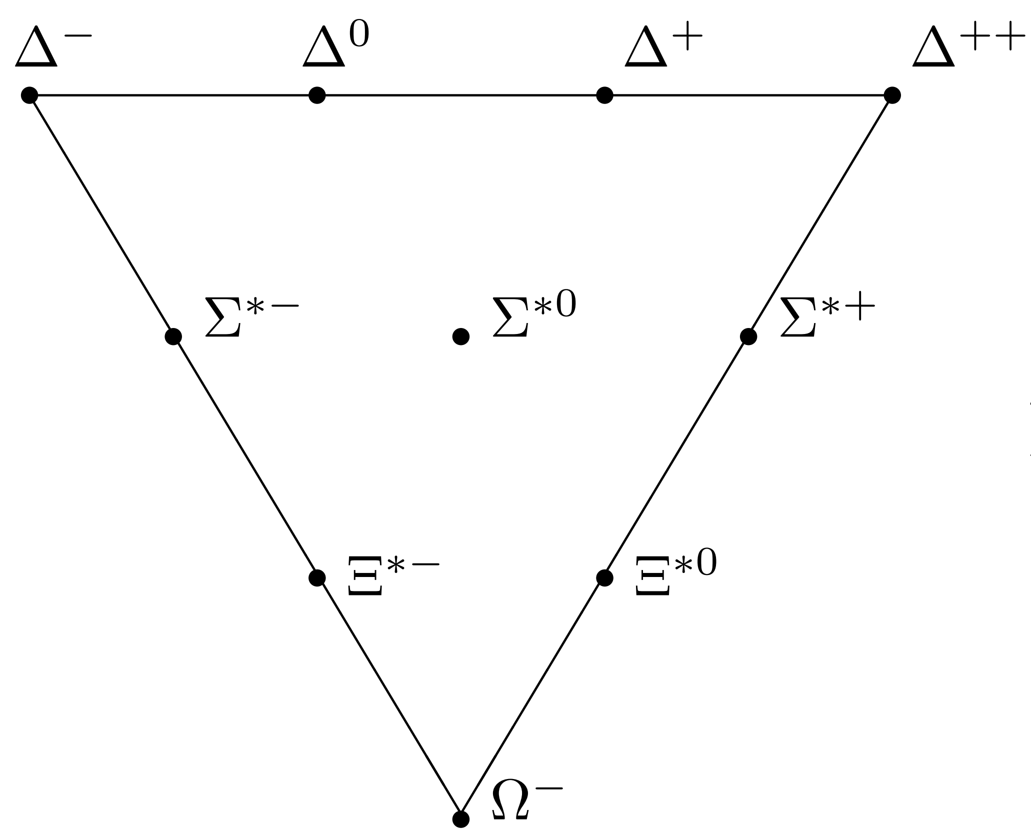
Section 4: Baryons

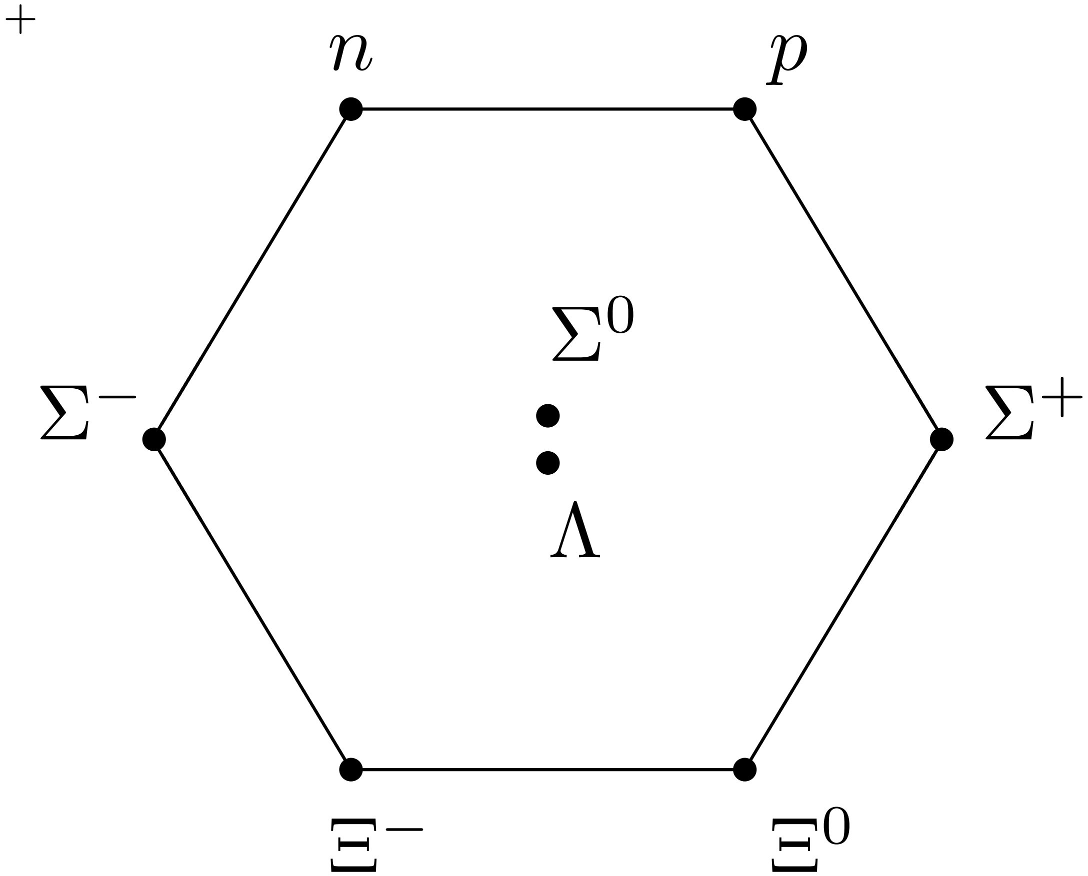
Mostly highlights from unpublished preliminary work in conference proceedings - (1610.06465)

Hyperons in SU(3)

Warm-up for SU(4) baryons



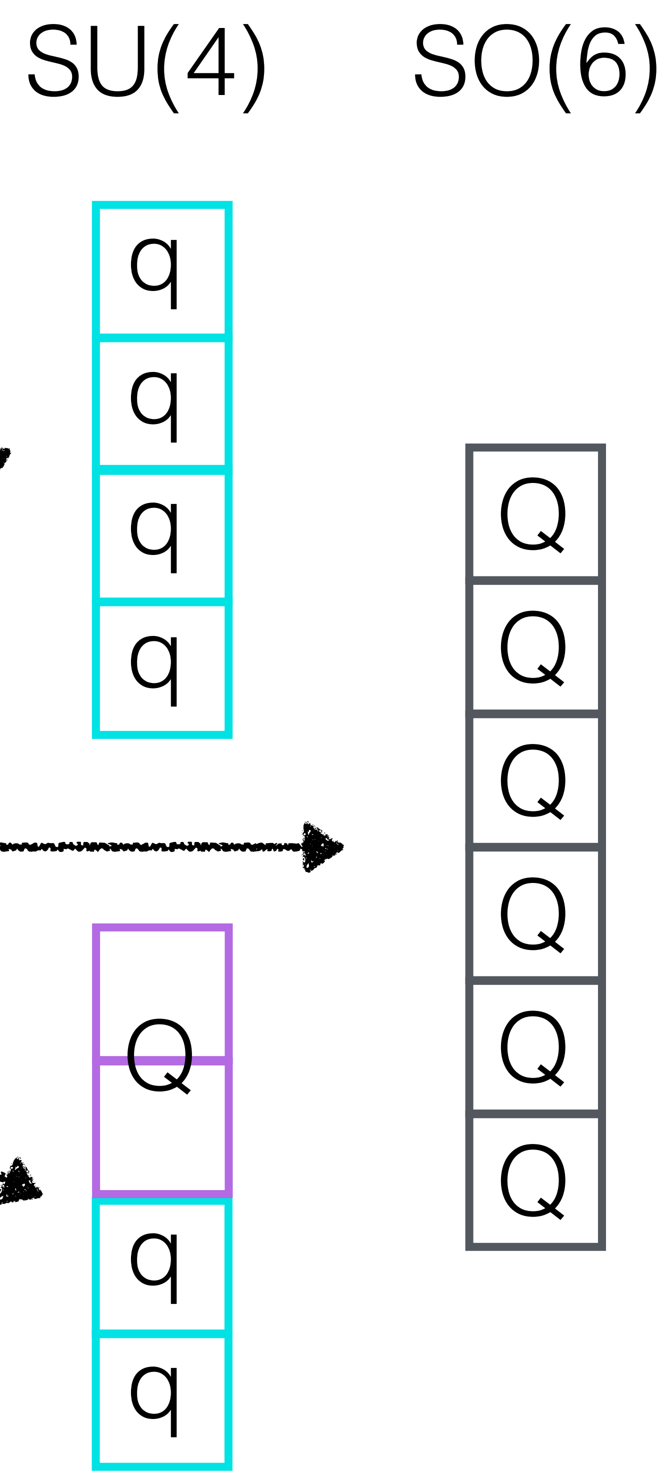




$$\begin{aligned} \Sigma^*(1390) &: I(J^P) = 1(3/2^+) \\ \Sigma(1190) &: I(J^P) = 1(1/2^+) \\ \Lambda(1120) &: I(J^P) = 0(1/2^+) \end{aligned}$$

Λ (isosinglet) = lightest QCD hyperon

Baryons in a multirep theory



- Three sorts of baryons appear in our model
  - Fundamental: $(qqqq)_{SU(4)}$
  - Sextet: $(QQQQQQ)_{SO(6)}$
  - Mixed or "chimera": $(Qqq)_{SU(4)}$

Baryon masses in multirep SU(4)

Goal: Qualitative understanding

- Tool: non-relativistic quark model
  - "Constituent" quark masses with "color hyperfine" interactions
  - NR quark models make quantitative predictions for the entire spectrum of multirep SU(4) baryons

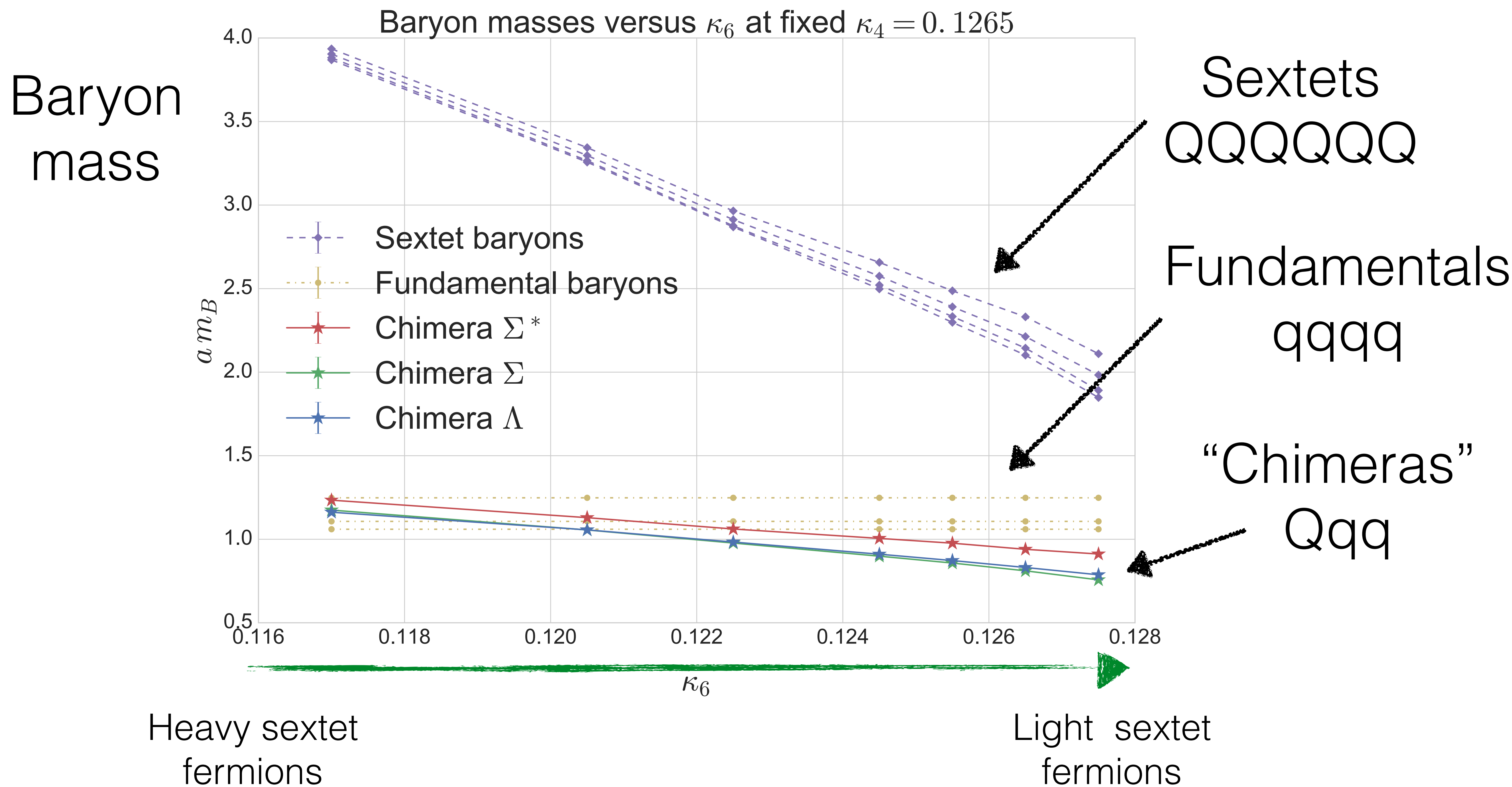
$$m_{qqqq} = 4m_q + \frac{C}{m_q^2} \sum_{i<j} \vec{S}_i \cdot \vec{S}_j = 4m_q + \frac{C}{2m_q^2} \left(\vec{S}_{\text{tot}}^2 - 3 \right)$$

$$m_{Qqq} = m_Q + 2m_q + \frac{C}{m_q^2} \left(\vec{S}_1 \cdot \vec{S}_2 + 2\frac{m_q}{m_Q} \vec{S}_Q \cdot (\vec{S}_1 + \vec{S}_2) \right)$$

- More generally: Dashen, Jenkins, and Manohar used SU(2)×U(1) flavor symmetry to derive similar functions at large-N

Exploratory results

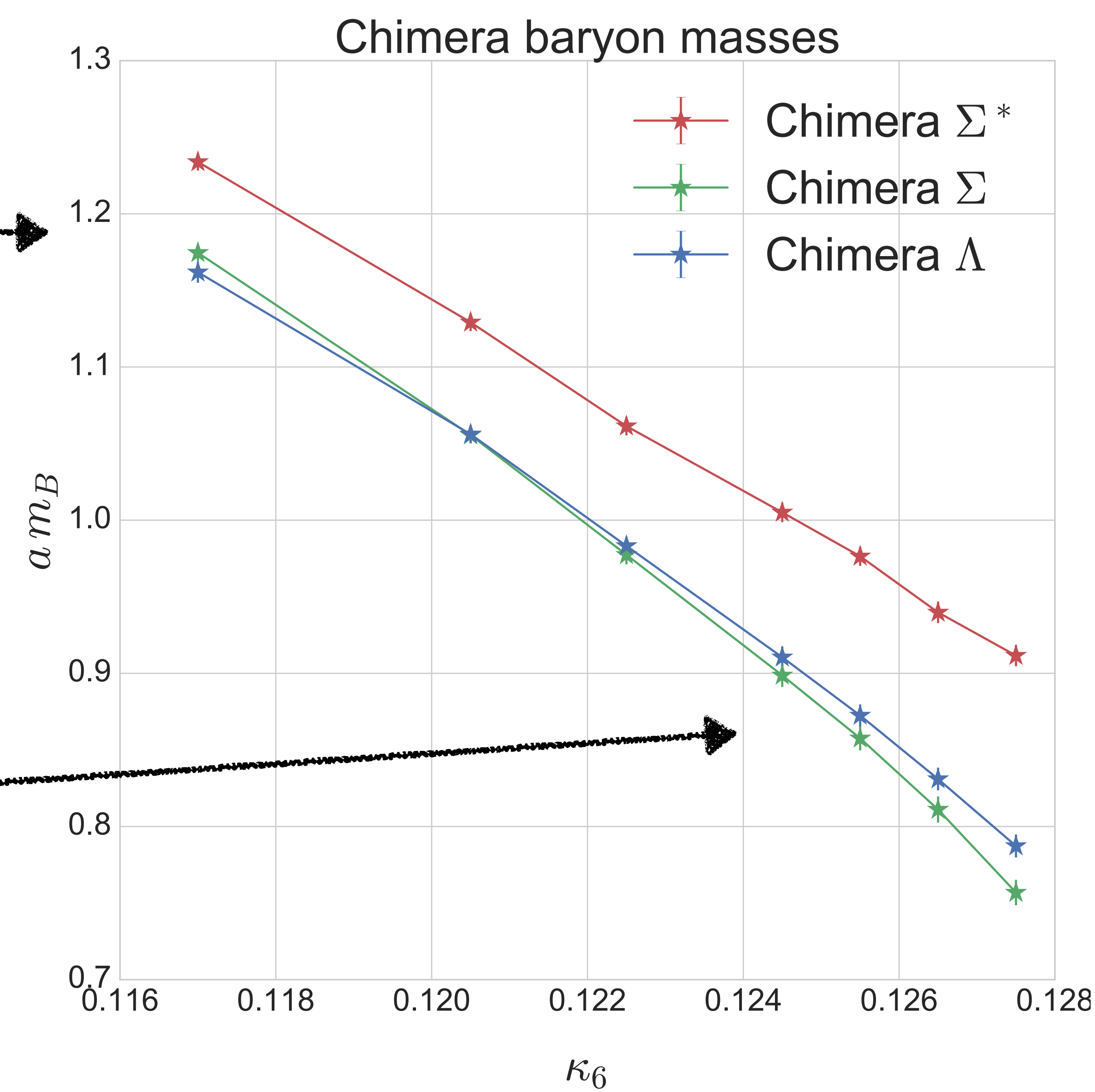
(1610.06465) — Single ensemble, no dynamical sextets



Exploratory results

(1610.06465) — Single ensemble, no dynamical sextets

“QCD-like region”
“Heavy” sextet fermion
$M_\Lambda < M_\Sigma$



Inverted region
“Light” sextet fermion
$M_\Lambda > M_\Sigma$

Summary, Conclusions

Summary

Meson spectroscopy

- ChiPT describes the Goldstone system well
    - Symmetries break as expected (the EFT works!)
    - The system is QCD-like, since the EFT is a close cousin of ChiPT in QCD
- Evidence exists for NLO communication between the two irreps
    - A qualitatively new phenomenon
    - An essential feature of any multirep theory
- The vector mesons are heavy states in this theory and potential targets for collider searches
    - KSRF relations suggest that such resonances are broad yet perhaps narrower than the $\rho(770)$ in QCD
    - Scaling from “large-N” helps explain the relative sizes between irreps

Summary
Higgs potential, Baryons

- The Higgs potential is generated via interactions with
  - EW gauge bosons: analogous to EM mass splittings among pions in QCD.
    - We computed the associated LEC in a single-rep simulation and plan to measure it on our multirep lattices
  - Top quark: baryon 4-point function, probably hard, although potentially tractable factorization at large-N
- For baryons, exploratory work with partial quenching suggests that
  - In multirep SU(4), baryons are well described by a NR quark model
  - Depending on the fermion masses, either the Σ-like or the Λ-like state can be the lightest state
  - The chimera baryons can be the lightest baryons in the spectrum — good news for phenomenology?
  - How much of this story will survive in a fully dynamical multirep simulation? (In progress, but very preliminary)

Take-home points

- You heard about preliminary results from simulations of SU(4) gauge theory with dynamical fermions in the fundamental and sextet representations
- This theory is a close relative of Ferretti's model of composite Higgs and partial compositeness
- The lattice can augment work in phenomenology by computing LECs, masses, etc…
- Meson spectroscopy is consistent with expectations from EFT

Future directions

- Thermodynamics of multirep theories (in progress)
    - Do representations condense and break chiral symmetry at different scales?
    - What is the nature of the transitions?
- Baryon spectroscopy on dynamical multirep lattices (in progress)
    - What is Γ_T/M_T for the top partner T?
    - Where does the top partner sit compared to the rest of the spectrum?
- Higgs potential
    - EW gauge boson contribution: repeat measurement in the full multirep theory
    - Top quark contribution: probably hard on the lattice (baryon 4-point functions?), but large-N estimates may help

Back-up slides

Ferretti's Model
Group theory details

- Overall: $G_F \to H_F = SU(3)_{diag} \times SU(2)_L \times SU(2)_R \times U(1)_X = G_{cust.} \supset G_{SM}$

- The global symmetry group is $G_F = SU(5) \times SU(3) \times SU(3)'$

  - χSB for the sextets: $SU(5) \to SO(5) \supset SO(4)$ "$\cong$" $SU(2)_L \times SU(2)_R$

  - χSB for the fundamentals: $SU(3) \times SU(3)' \to SU(3)_{diag} \times U(1)_X$

  - The Higgs lives in the coset $SU(5)/SO(5)$

Slide credit: Ethan Neil

Software: Multirep MILC

- Based on a branch of the MILCv7 code, focused on Wilson fermions
- Dynamical code generation using Perl: N_c and representation(s) are fixed in code generation, allowing the C compiler to optimize matrix operations
- Bells and whistles: clover term, nHYP smearing, Hasenbusch preconditioning, multi-level integrator, NDS action, …
- We use all of the above in our simulations. The clover term c_{SW} is set equal to unity (shown to work well with smearing)

The NDS Action

nHYP Dislocation Suppressing Action

- nHYP is a smearing scheme invented and optimized by Hasenfratz and Knechtli. It involves fat links V built from thin links U.
  - The usual gauge links U are “thin” links. The fat link V is “smeared” link — a sum of products of gauge links connecting points on the lattice.
  - Smearing provides a smoother background for fermion propagation. Smoothing is known to reduce lattice artifacts.
- Dislocation suppression refers to taming large spikes in the fermion force during HMC evolution.
  - Enacted by extra marginal gauge terms
  - Creates a “repulsive potential” to cancel out the offending large spikes in the fermion force.

Lattice Spectroscopy

- Two-point functions encode spectral information, as usual
  - The axial Ward identity yields the quark mass

$$\partial_\mu \langle 0|A^\mu(x)\mathcal{O}|0\rangle = 2m_q \langle 0|(\overline{\psi}\gamma^5\psi)_x\mathcal{O}|0\rangle$$

  - Decay constants use "130 MeV" conventions (and its natural generalization)

$$\langle 0|\overline{u}\gamma^\mu\gamma^5 d|\pi(p)\rangle = iF_\pi p^\mu$$

- Many possible ways to set the scale
  - Sommer parameters r_0, r_1
  - The flow scale t_0
  - Mass of the Ω-baryon, decay constants F_π or $F_{K,\ etc\ldots}$
  - Safe flow scales? $1 < t_0 / a^2 < 3$, "QCD" analogy: 0.08 fm $\lesssim$ a $\lesssim$ 0.13 fm

Decay Constants

Normalization and Conventions

- Decay constants with Wilson fermions involve a rescaling factor which depends on the critical value of the hopping parameter κ_{critical}.
  - For these ensembles, $\kappa \sim \kappa_{\text{critical}}$.
  - The Wilson normalization term does not vary much across the ensembles
- Decay constants also involve a (perturbative) matching factor, Z
  - For these ensembles, the Z-factors were approximately unity

$$F_P \sim (\text{Z-factor}) \times (\text{Wilson-}\kappa_{\text{critical}} \text{ factor}) \times F_{P,\text{raw}}$$

ChiPT at NLO — $M_P{}^2$

$$
\begin{aligned}
\hat{M}_{P4}^2 =& 2\hat{m}_{q4}\hat{B}_4\left[1 + L_{44}\hat{m}_{q4} + L_{46}\hat{m}_{q6} + \frac{1}{2}\Delta_4 - \frac{4}{5}\Delta_\zeta\right] \\
&+ A_{\mathrm{art}}^{M4} am_{q4} + B_{\mathrm{art}}^{M4} am_{q6} + C_{\mathrm{art}}^{M4}\frac{a^2}{t_0} \\
\hat{M}_{P6}^2 =& 2\hat{m}_{q6}\hat{B}_6\left[1 + L_{66}\hat{m}_{q6} + L_{64}\hat{m}_{q4} - \frac{1}{4}\Delta_6 - \frac{1}{5}\Delta_\zeta\right] \\
&+ A_{\mathrm{art}}^{M6} am_{q4} + B_{\mathrm{art}}^{M6} am_{q6} + C_{\mathrm{art}}^{M6}\frac{a^2}{t_0}
\end{aligned}
$$

ChiPT at NLO — F_P

$$\hat{F}_{P4} = \hat{F}_4 \left[1 + C_{44}\hat{m}_{q4} + C_{46}\hat{m}_{q6} - \Delta_4\right] + C^{F4}_{\mathrm{art}} \frac{a}{\sqrt{t_0}}$$

$$\hat{F}_{P6} = \hat{F}_6 \left[1 + C_{66}\hat{m}_{q6} + C_{64}\hat{m}_{q4} - 2\Delta_6\right] + C^{F6}_{\mathrm{art}} \frac{a}{\sqrt{t_0}}$$

ChiPT at NLO — Chiral Logs

$$\Delta_4 = \frac{2\hat{m}_{q4}\hat{B}_4}{8\pi^2\hat{F}_4^2}\log\left[2\hat{m}_{q4}\hat{B}_4\right]$$

$$\Delta_6 = \frac{2\hat{m}_{q6}\hat{B}_6}{8\pi^2\hat{F}_6^2}\log\left[2\hat{m}_{q6}\hat{B}_6\right]$$

$$\Delta_\zeta = \frac{\hat{M}_\zeta^2}{8\pi^2\hat{F}_\zeta^2}\log\left[\hat{M}_\zeta^2\right]$$

$$\hat{M}_\zeta^2 = \frac{8}{5}\left(\frac{2\hat{F}_4^2\hat{m}_{q4}\hat{B}_4 + \hat{F}_6^2\hat{m}_{q6}\hat{B}_6}{F_\zeta^2}\right)$$

Vectors: Empirical models

$$\hat{M}_{V,r} = p_0 + p_1 \hat{m}_{q,r} + p_2 \hat{m}_{q,r}^2 + p_3 \hat{m}_{q,r} \hat{m}_{q,\tilde{r}} + A_{\mathrm{art}} m_{q,r} a + B_{\mathrm{art}} \frac{a}{\sqrt{t_0}} + C_{\mathrm{art}} m_{q,\tilde{r}} a + D_{\mathrm{art}} \frac{a^2}{t_0},$$

And similar for the vector decay constants…
The parameters p_i are unrelated among the four vector quantities

Modelling M_V and F_V